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UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

PART II—A TRANSONIC BOX METHOD FOR PLANAR LIFTING SURFACES

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NOTICES

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FOREWORD

This part of the report covers a portion of the research conducted by the Space and Information Systems Division of North American Aviation, Inc., Downey, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF 33(657)-10399.

The work was performed to advance the state of the art of flutter prediction for flight vehicles as part of the Research and Technology Division, Air Force Systems Command's exploratory development program. This research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicle," and Task No. 137003 "Prediction and Prevention of Aerothermoelastic Problems." Mr. James Olsen of the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, was the Project Engineer.

Mr. L. V. Andrew was the Program Manager for North American Aviation. Dr. E. R. Rodemich developed the technical approach and wrote the computer programs. Several valuable suggestions were given by Dr. M. T. Landahl of the Massachusetts Institute of Technology.

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This report has been reviewed and is approved.

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ABSTRACT

The fundamental equations of the transonic box method were derived, based on the representation of the velocity potential by a doublet distribution. They form the basis of a systematic method of treating an oscillating wing at M = 1, analogous to the supersonic Mach box method.

A digital computer program, written in Fortran IV, is presented. The program applies to a planar wing of polygonal planform, with a straight trailing edge, and as many as three sweep angles along the leading edge. For a maximum of ten modes of oscillation, the program computes the oscillatory potentials and pressures and a generalized force matrix.

Results obtained from the program are compared with existing theoretical and experimental values. Several possible extensions of the method are described.

TABLE OF CONTENTS

ILLUSTRATIONS

LIST OF SYMBOLS

Symbol	Definition
a	Local speed of sound; speed of sound at infinity
a_{nm}	Coefficient in the potential series
$A(i - i', \|j - j'\|)$	Influence coefficient: the upwash at the center of B_{ij} caused by a unit doublet distribution over $B_{i'j'}$
A_{jr}	Term in $\bar{\varphi}$ evaluated at (x_j, y_j)
AXY (I, J)	An integral over the wing planform
AY (J)	An integral along the trailing edge
b	Root chord length
B	Region composed of boxes, approximating S
B_{ij}	A box
$(B_{rr'})$	Matrix used in least squares surface fits
BXY	Part of AXY (I, J)
BY	Part of AY (J)
$\bar{C}_p$	Pressure coefficient
(C'_r), (C''_r)	Column matrices used in least squares surface fits
d	Dimensionless length of box side
d_{nm}	Coefficient in deflection polynomial
DA	The data array
f	Function which describes the wing deflection
F	Factor which gives $\bar{\varphi}$ the proper edge behavior

(Continued on next page.)

Symbol	Definition
$\tilde{g}_u$, $\tilde{g}_\ell$, $\tilde{f}$	Functions used in the equations of upper and lower wing surfaces
h_j	Weight used in Gaussian quadrature
i	$\sqrt{-1}$
i, j	Indexes specifying box position
I, J	Indexes
k	Reduced frequency: $\omega b/U_\infty$
ℓ	kd
L_{ij}	Generalized force coefficient
M	Mach number
n, m	Indexes equal to power of x and power of y^2
NC	Number of coefficients
NP	Number of points
NS	Number of segments of leading edge given by the data
p, q	Integration variables
Q	Quantity minimized in least squares surface fits
r	Index
S	Function used in the equation of a surface
S	Region in the xy-plane occupied by the wing; the area of this region
t	Time
u, v	Integration variables
u_j	Point used in Gaussian quadrature
U_∞	Air speed of the wing; speed of flow at infinity

(Continued on next page.)

Symbol	Definition
w	Upwash at $z = 0+$
W	The region of the xy-plane occupied by the wing's wake
$\tilde{x}, \tilde{y}, \tilde{z}$	Coordinates with dimensions of length
x, y, z	Dimensionless coordinates
(x_i, y_j)	Center of B_{ij}
(x_j, y_j)	Point at which a value of potential or deflection is given
$x_1, \ldots, x_{NS}$, $y_o, \ldots, y_{NS}$	Coordinates of points on the leading edge given by the data
x_o	Function which describes the leading edge: $x = x_o(y)$
y_{max}	Value of y at the wing tip
y_+, y_-	Limits of integration
α_{nm}, α_r	Real part of a_{nm}
β_r	Imaginary part of a_{nm}
δ	Constant factor in the deflection
Δp_i	Lifting pressure in the ith mode
ξ, η	Integration variables equivalent to x, y
ν	Frequency
ρ	Density
ρ	Source or doublet strength
σ	The integral over x involved in BXY
Φ	Velocity potential
ϕ	Steady perturbation potential
φ	Unsteady perturbation potential

(Continued on next page.)

Symbol	Definition
$\bar{\varphi}$	Time independent factor of φ
$\bar{\varphi}_o$	Potential of a point source
$\bar{\varphi}_1$	Potential of a point doublet
$\bar{\varphi}_s$	Potential of a source distribution
$\bar{\varphi}_d$	Potential of a doublet distribution
$\bar{\varphi}_{ij}$	Value of $\bar{\varphi}$ in B_{ij}
$\bar{\varphi}'_j$	Real part of value of $\bar{\varphi}$ at (x_j, y_j)
Ψ	Upwash in the xy-plane caused by a point doublet
ω	Angular frequency, $2\pi\nu$

1. INTRODUCTION

The transonic box program is designed to calculate the unsteady potentials for a given set of modes of wing oscillation and to compute the generalized forces. Pressure distributions may be obtained from the potentials.

A planar wing with a straight trailing edge is assumed. The oscillations are assumed to be symmetric in the spanwise coordinate y. None of these assumptions is necessary for the method. (See Section 5.)

The basic step in the box method is the solution of the system of simultaneous equations [Equation (33)] which determine a set of values of potential on the wing from a corresponding array of upwash values. A surface is fitted to these values, giving a functional representation of the potential that is used subsequently to find pressures and generalized forces.

The method used is suggested by the success of supersonic box methods (References 1 through 4). The potential is generated by a doublet distribution rather than by a source distribution because the latter method would involve diaphragm regions of infinite extent, whereas the doublet distribution is confined to the wing and its wake.

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2. THEORETIC... DEVELOPMENT OF THE METHOD

1. THE DIFFERENTIAL EQUATION

We consider an oscillating body moving at speed U_∞ through a nonviscous fluid. From the point of view of a moving coordinate system $(\bar{x}, \bar{y}, \bar{z})$ in which the average position of the body is fixed, there is a flow past the body with velocity U_∞ at infinity. Assume that the flow is irrotational; then the velocity field of the flow is the gradient of a potential function Φ, which satisfies the differential equation

$$\nabla^2 \Phi - \frac{1}{a^2}\left[\Phi_{tt} + 2\nabla\Phi \cdot \nabla\Phi_t + (\nabla\Phi \cdot \nabla)\, 1/2\, (\nabla\Phi)^2\right] = 0 \qquad (1)$$

(See Reference 5, p. 193) where a is the local speed of sound.

Suppose that the flow is approximately uniform in the direction of the positive $\bar{x}$-axis. This may be true, for example, if the body is almost plane and the oscillations are small. Then Φ may be broken up into several parts, as

$$\Phi = U_\infty(\bar{x} + \phi + \varphi) \qquad (2)$$

where the first term gives a uniform flow, the second term gives the correction for a steady flow about the body, the third term gives the correction to this for the oscillating body, and ϕ and φ are small.

To the first order, ϕ and φ are different solutions of the same differential equation

$$(1 - M^2)\, \varphi_{\bar{x}\bar{x}} + \varphi_{\bar{y}\bar{y}} + \varphi_{\bar{z}\bar{z}} - \frac{2\, U_\infty}{a^2}\varphi_{\bar{x}t} - \frac{1}{a^2}\varphi_{tt} = 0 \qquad (3)$$

where M, a are the Mach number and speed of sound at infinity. (See Reference 5, p. 198.) φ is a periodic function of t. Since the differential equation is linear, we may put $\varphi = \bar{\varphi}(x, y, z)\, e^{i\omega t}$, where ω is the angular frequency of oscillation. In terms of the nondimensional quantities,

$$x = \tilde{x}/b$$
$$y = \tilde{y}/b$$
$$z = \tilde{z}/b$$
$$k = \omega b/U_\infty$$

(b is a characteristic length of the body); Equation (3) becomes

$$(1-M^2)\bar{\varphi}_{xx} + \bar{\varphi}_{yy} + \bar{\varphi}_{zz} - 2iM^2k\bar{\varphi}_x + M^2k^2\bar{\varphi} = 0 \tag{4}$$

For M = 1, this reduces to

$$\bar{\varphi}_{yy} + \bar{\varphi}_{zz} - 2ik\bar{\varphi}_x + k^2\bar{\varphi} = 0 \tag{5}$$

the linearized transonic equation (see Reference 6, p. 7). It has been suggested by Landahl (Reference 6) that the proper equation to use instead of (4) is

$$\bar{\varphi}_{yy} + \bar{\varphi}_{zz} - 2iM^2k\bar{\varphi}_x + M^2k^2\bar{\varphi} = 0$$

if $k \gg |M-1|$. Comparison of this equation with (5) leads to a similarity rule for flows in the transonic range (see Reference 6, p. 18).

The range of validity of this equation is duscussed by Landahl (Reference 6, Chapter 1). First, there is the requirement for linearization in any speed range, that the perturbation potential $\phi + \varphi$ be small. This is not satisfied at the leading edge of a wing for any realistic cross-sectional shape; however, it may be satisfied over the rest of the wing, if the wing has small thickness, and the results on parts of the wing away from the leading edge are not much affected by the error there.

Another restriction peculiar to transonic speeds is associated with the absence of the term in $\bar{\varphi}_{xx}$. The actual flow has some variation in local Mach number which may influence the nature of the flow considerably if M is near 1. The presence of the term in $\bar{\varphi}_x$ tends to reduce this influence, but for k small or zero, the equation is valid only for a highly swept wing with a pointed nose.

The difference of the local Mach number from the value 1 assumed in Equation (5) may come from two sources: (1) wing thickness, and (2) a change in the free stream Mach number. Thus, for any value of k, there are limits on the thickness ratio and the Mach number range, which increase with k. Estimates of these limits are not possible, because of the small amount of experimental data available.

2. BOUNDARY CONDITIONS

The solution of Equation (1) must give a velocity field which is such that a particle at the body surface moves along the moving surface. If the equation of the surface is

$$S(\tilde{x}, \tilde{y}, \tilde{z}, t) = 0$$

this equation must be satisfied when $(\tilde{x}, \tilde{y}, \tilde{z})$ moves with the velocity $\nabla\Phi$. Differentiating with respect to t gives the condition

$$\nabla\Phi \cdot \nabla S + \frac{\partial S}{\partial t} = 0 \qquad (6)$$

This determines the normal velocity at the surface.

Now suppose that the body (to be referred to henceforth as a wing) is almost planar, lying almost in the xy-plane (see Figure 1). For vertical

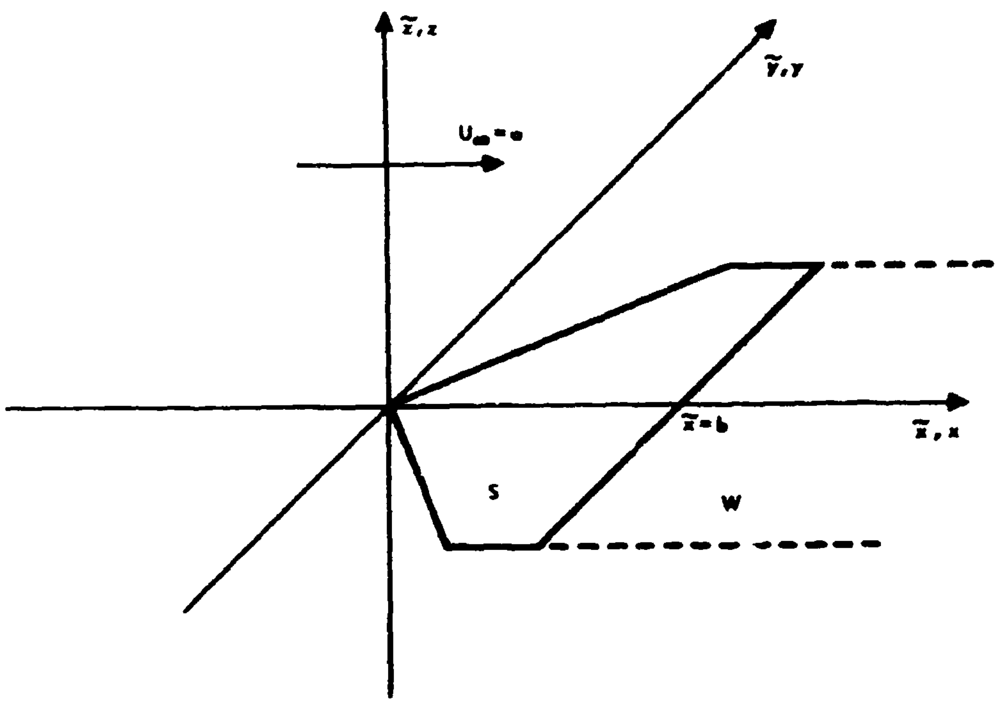


Figure 1. Coordinate Systems

oscillations of the body, the upper and lower surfaces may be represented by the equations

$$\tilde{z} = \tilde{g}_u (\tilde{x}, \tilde{y}) + e^{i\omega t} \tilde{f} (\tilde{x}, \tilde{y})$$

$$\tilde{z} = \tilde{g}_{\ell}(\tilde{x}, \tilde{y}) + e^{i\omega t}\tilde{f}(\tilde{x}, \tilde{y})$$

where the functions $\tilde{g}_u, \tilde{g}_{\ell}$ are associated with the deviation of the shape of the body from planar, and $\tilde{f}$ depends on the mode of oscillation. Then on the two surfaces, we may take

$$S = \tilde{z} - \tilde{g}_u - e^{i\omega t}\tilde{f}$$

$$S = \tilde{z} - \tilde{g}_{\ell} - e^{i\omega t}\tilde{f}$$

Use these expressions for S and Equation (2) in Equation (6). Neglecting terms that involve products of φ or ϕ with $\tilde{g}_u, \tilde{g}_{\ell}$, or $\tilde{f}$, the resulting equation may be broken up into a steady part, which gives the boundary condition for ϕ, and an unsteady part, which gives the boundary condition for $\bar{\varphi}$. The unsteady part is

$$\frac{\partial \bar{\varphi}}{\partial z} = \frac{\partial f}{\partial x} + ikf \tag{7}$$

where $f = \tilde{f}/b$. To the present degree of approximation, this condition should be applied at $z = 0$, over the region of the xy-plane on which the body projects.

3. THE BOUNDARY VALUE PROBLEM FOR $\bar{\varphi}$

In linearized theory, a disturbance of a flow at Mach 1 does not have any influence upstream. Consequently,

$$\bar{\varphi}(x, y, z) = 0, \quad x < 0 \tag{8}$$

if the body lies in the region $x \geq 0$. This is one of the conditions $\bar{\varphi}$ must satisfy.

$\bar{\varphi}$ is a solution of Equation (5) in all space outside S and W, the regions in the xy-plane occupied by the wing and its wake (see Figure 1). In general, $\bar{\varphi}$ is discontinuous in these regions. A boundary condition on W is obtained by equating the pressures above and below the surface of the wake. From the linearized form of the pressure coefficient,

$$\bar{C}_p = -2(\bar{\varphi}_x + ik\bar{\varphi})$$

(see Reference 6, p. 15) we get

$$\left[\bar{\varphi}_x(x,y,z) + ik\bar{\varphi}(x,y,z)\right]\Bigg|_{z=0-}^{0+} = 0, \quad (x,y) \text{ in } W \tag{9}$$

This condition, plus Equation (7) applied on the two sides of S, plus Equation (8), determine $\bar{\varphi}$ as a solution of Equation (5).

The conditions satisfied by $\bar{\varphi}(x,y,z)$ are satisfied also by $-\bar{\varphi}(x,y,-z)$. Hence, $\bar{\varphi}$ is an odd function of z. This implies that $\bar{C}_p$ is zero in the wake. In the half space $z > 0$, $\bar{\varphi}$ is a solution of Equation (5), which satisfies Equation (8) and the boundary conditions

$$\bar{\varphi}_z(x,y,0+) = \frac{\partial f}{\partial x} + ikf, \quad (x,y) \text{ in } S \tag{10}$$

$$\bar{\varphi}_x(x,y,0+) + ik\bar{\varphi}(x,y,0+) = 0, \quad (x,y) \text{ in } W \tag{11}$$

$$\bar{\varphi}(x,y,0+) = 0, \quad (x,y) \text{ not in } S + W \tag{12}$$

Such a solution may be built up from a doublet distribution over $S + W$ or a source distribution over the half plane $z = 0$, $x > 0$.

4. BASIC SOURCE AND DOUBLET SOLUTIONS OF THE DIFFERENTIAL EQUATION (See References 7, 8, and 9.)

The solution of Equation (5) which represents a point source at the origin is

$$\bar{\varphi}_o(x,y,z) = \begin{cases} 0, & x \le 0 \\ -\dfrac{1}{2\pi}\dfrac{1}{x}\, e^{-\frac{1}{2}ik\left(x + \frac{y^2+z^2}{x}\right)}, & x > 0 \end{cases} \tag{13}$$

(See Reference 9.) The potential of a point doublet oriented parallel to the z-axis is obtained by differentiation, as

$$\bar{\varphi}_1(x,y,z) = \frac{\partial\bar{\varphi}_o}{\partial z} = \begin{cases} 0, & x \le 0 \\ \dfrac{ik}{2\pi}\dfrac{z}{x^2}\, e^{-\frac{1}{2}ik\left(x + \frac{y^2+z^2}{x}\right)}, & x > 0 \end{cases} \tag{14}$$

It is easily verified that these functions satisfy Equation (5) for $x \neq 0$. They are poorly behaved at $x = 0$ for real values of k.

To improve the behavior of $\bar{\varphi}_0$ and $\bar{\varphi}_1$ at $x = 0$, assume that k has a small negative imaginary part. This causes $\bar{\varphi}_0$ and $\bar{\varphi}_1$ to approach zero exponentially as $x \to 0+$, except at the origin. All partial derivatives of all orders have the same property. Thus, $\bar{\varphi}_0$ and $\bar{\varphi}_1$ are solutions of Equation (5) everywhere except at (0, 0, 0). In the final formulas to be obtained, the imaginary part of k can be put equal to zero.

Solutions of Equation (5) for $z > 0$ which satisfy Equation (8) are given for a distribution of sources as

$$\bar{\varphi}_s(x, y, z) = \iint\limits_{\xi > 0} \rho(\xi, \eta)\, \bar{\varphi}_0(x-\xi,\ y-\eta,\ z)\, d\xi\, d\eta \tag{15}$$

and for a distribution of doublets as

$$\bar{\varphi}_d(x, y, z) = \iint\limits_{\xi > 0} \rho(\xi, \eta)\, \bar{\varphi}_1(x-\xi,\ y-\eta,\ z)\, d\xi\, d\eta \tag{16}$$

where, to be completely general, $\rho(\xi, \eta)$ may be any function such that the integrals exist. From the form of $\bar{\varphi}_0$ and $\bar{\varphi}_1$, the region of integration may be restricted to the plane strip $0 < \xi < x$. It is shown in Appendix I that these functions satisfy the following boundary conditions for $z = 0$, $x > 0$:

$$\bar{\varphi}_{sz}(x, y, 0+) = \rho(x, y) \tag{17}$$

$$\bar{\varphi}_d(x, y, 0+) = \rho(x, y) \tag{18}$$

(in fact, if the same function ρ is used in both integrals, $\bar{\varphi}_d = \partial\bar{\varphi}_s/\partial z$).

5. THE DETERMINATION OF $\bar{\varphi}$ BY A SOURCE DISTRIBUTION

One method of attack on the problem of finding $\bar{\varphi}$ is to set $\bar{\varphi} = \bar{\varphi}_s$. Then, in terms of the upwash

$$w(x, y) = \bar{\varphi}_z(x, y, 0+) \tag{19}$$

we have from Equations (17) and (15)

$$\bar{\varphi}(x, y, z) = \iint\limits_{\xi > 0} w(\xi, \eta)\, \bar{\varphi}_0(x-\xi,\ y-\eta,\ z)\, d\xi\, d\eta \tag{20}$$

for $z \geq 0$.

The values of w on S are known by Equation (10). Elsewhere, w is unknown, and it must be chosen so that the boundary conditions (11) and (12) are satisfied. We may take the limit as $z \to 0+$ in Equation (20) by taking the limit under the integral sign:

$$\bar{\varphi}(x, y, 0+) = \iint\limits_{\xi > 0} w(\xi, \eta)\, \bar{\varphi}_o(x-\xi,\ y-\eta,\ 0)\, d\xi\, d\eta \tag{21}$$

From Equations (11) and (12) are obtained the system of integral equations

$$\iint\limits_{\xi > 0} w(\xi, \eta)\, \bar{\varphi}_o(x-\xi,\ y-\eta,\ 0)\, d\xi\, d\eta = 0, \quad (x, y) \text{ not in } S + W \tag{22}$$

$$\left(\frac{\partial}{\partial x} + ik\right) \iint\limits_{\xi > 0} w(\xi, \eta)\, \bar{\varphi}_o(x-\xi,\ y-\eta,\ 0)\, d\xi\, d\eta = 0, \quad (x, y) \text{ in } W \tag{23}$$

Solution of Equations (22) and (23), followed by evaluation of $\bar{\varphi}$ according to Equation (21), would yield the values of $\bar{\varphi}$ on S, from which pressures and forces can be computed.

A box method based on a source distribution, described briefly in Reference 9, has been used by Weatherill at the Boeing Company. Some of his preliminary results are given in Reference 9.

6. THE DETERMINATION OF $\bar{\varphi}$ BY A DOUBLET DISTRIBUTION

If we set $\bar{\varphi} = \bar{\varphi}_d$, then by Equations (18), (16), and (12)

$$\bar{\varphi}(x, y, z) = \iint\limits_{S+W} \bar{\varphi}(\xi, \eta, 0+)\, \bar{\varphi}_1(x-\xi,\ y-\eta,\ z)\, d\xi\, d\eta \tag{24}$$

In terms of

$$\psi(x, y) = \lim_{z \to 0} \frac{1}{z} \bar{\varphi}_1(x, y, z) = \begin{cases} 0, & x \le 0 \\[2ex] \dfrac{ik}{2\pi} \dfrac{1}{x^2}\, e^{-\frac{1}{2} ik \left(x + \frac{y^2}{x}\right)}, & x > 0 \end{cases} \tag{25}$$

the normal derivative of $\bar{\varphi}$ at $z = 0$ is given by a singular integral:

$$w(x, y) = \iint_{S+W} \bar{\varphi}(\xi, \eta, 0+)\, \psi(x-\xi, y-\eta)\, d\xi\, d\eta \tag{26}$$

The values of $\bar{\varphi}$ $(\xi, \eta, 0+)$ must be determined then from

$$\iint_{S+W} \bar{\varphi}(\xi, \eta, 0+)\, \psi\, (x-\xi, y-\eta)\, d\xi\, d\eta = w(x, y), \quad (x, y) \text{ in } S \tag{27}$$

$$\left(\frac{\partial}{\partial x} + i k\right) \bar{\varphi}(x, y, 0+) = 0, \quad (x, y) \text{ in } W \tag{28}$$

7. A COMPARISON OF THE METHODS

Except for the singularity of the integral in Equation (27), all points of difference are in favor of solving the problem by doublets. There are these points:

a. The region of integration in the source method extends theoretically to $\pm\infty$ in η; even practically, the region must be extended an extreme distance. In the doublet method, the region is restricted to S + W. This distinction is not so great for supersonic flows. There, the region of influence of the wing is swept back along Mach lines, and the set of points in this region that influences the wing is bounded (see Reference 10).

b. After the unknown function under the integral sign is known, the source method requires an extra step – the evaluation of $\bar{\varphi}$ on the wing from Equation (21).

c. If values in the wake must be considered, the condition in the wake for the source method, Equation (23), is more complicated than the corresponding condition, Equation (28), for the doublet method.

The doublet method was used because of point a.

8. THE ADVANTAGE OF A STRAIGHT TRAILING EDGE

Suppose the wing has a straight trailing edge perpendicular to the direction of flow (x = constant along the edge); then the wing is not influenced by the wake. This is reflected in the equations by the fact that the integrands are zero when $\xi > x$. Hence, in either method, for the determination of $\bar{\varphi}$ on the wing, the condition in W need not be used.

9. THE DOUBLET BOX METHOD

Consider a flow at Mach 1 past an oscillating wing with its nose at the origin, lying approximately in the xy-plane, with x = 1 along the trailing edge. The value of the unsteady potential $\bar{\varphi}$ on the wing may be found by solution of Equation (27), which may be written as

$$\iint_S \bar{\varphi}(\xi, \eta, 0+)\, \psi(x-\xi, y-\eta)\, d\xi\, d\eta = w(x, y), \quad (x, y) \text{ in } S \qquad (29)$$

To get an approximate solution of this equation, let the xy-plane be covered with a grid of square boxes with sides of length d, so that box edges lie along the coordinate axes (see Figure 2). Let the region B be composed of all boxes whose centers lie in S; B is an approximation to S by boxes. Let i, j be box indexes in the x- and y-directions. Approximate $\bar{\varphi}$ by a constant value $\bar{\varphi}_{ij}$ in the (i, j)-th box B_{ij}. Impose the condition of Equation (7) at the center (x_i, y_j) of each box B_{ij} in B, with the region of integration replaced by B. Then Equation (29) gives a system of linear algebraic equations for the $\bar{\varphi}_{ij}$'s:

$$\sum_{i', j'} \bar{\varphi}_{i'j'} \iint_{B_{i'j'}} \psi(x_i-\xi, y_j-\eta)\, d\xi\, d\eta = w(x_i, y_j) \qquad (30)$$

Examination of the integral in Equation (30) shows that it depends on i, j, i', j' only via i-i', |j-j'|. The notation

$$A(i-i', |j-j'|) = \iint_{B_{i'j'}} \psi(x_i-\xi, y_j-\eta)\, d\xi\, d\eta \qquad (31)$$

is introduced. Formulas for the evaluation of this quantity are given in Appendix II.

Segregating the terms with i'=i on the left, Equation (30) becomes

$$\sum_{j'} A(0, |j-j'|)\, \bar{\varphi}_{ij'} = w(x_i, y_j) - \sum_{i'<i} \sum_{j'} A(i-i', |j-j'|)\, \bar{\varphi}_{i'j'} \qquad (32)$$

For fixed i and varying j, this is a smaller system of equations that may be solved for each consecutive value of i.

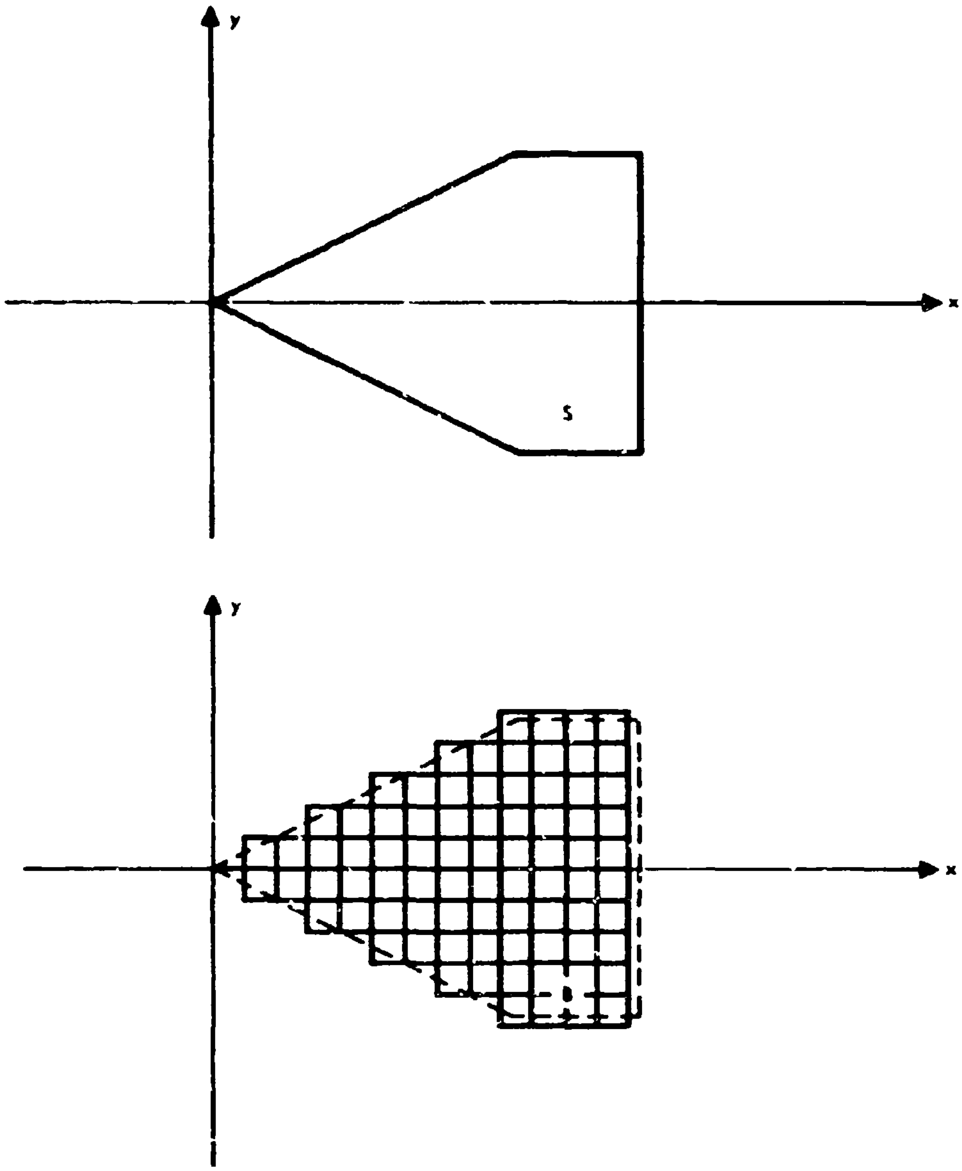


Figure 2. Approximation of the Wing by Region B

Now suppose the wing is symmetric about the x-axis; then only modes of oscillation that are symmetric or antisymmetric in y need be treated. Consider a symmetric mode. $\bar{\varphi}_{ij}$ will have the same value at corresponding boxes across the x-axis. This may be used to reduce the range of the sum in Equation (32) and the range of j. Let j = 1 in the row of boxes in which $0 < y < d$. Then, combining terms for symmetrically placed boxes,

$$\sum_{j' \geq 1} [A(0, |j-j'|) + A(0, j+j'-1)] \; \bar{\varphi}_{ij'} \tag{33}$$

$$= w(x_i, y_j) - \sum_{i'<i} \sum_{j' \geq 1} [A(i-i', |j-j'|) + A(i-i', |j-j'|)] \; \bar{\varphi}_{i'j'}, \; j \geq 1$$

The equations for $j \leq 0$ are implied by those with $j \geq 1$. Thus, the size of the system has been reduced by a factor of 2.

For antisymmetric modes, Equation (33) applies, with the sums of values of A replaced by differences.

10. EXTENSIONS OF THE METHOD

The computer program discussed in Section 3 has some restrictions that are not inherent in the box method, such as the requirement of a straight trailing edge. Some possible modifications that extend the applicability of the program will now be described.

To modify the program for modes antisymmetric in y, it is only necessary to change some of the signs in Equation (33), as indicated in the discussion above, and replace even powers of y by odd powers in the formulas used for deflection and potential.

To deal with a more general trailing edge, it is necessary to use the values of $\bar{\psi}$ in the wake. For fixed y, if $x = x_T$ at the trailing edge, Equation (28) may be integrated to give

$$\bar{\psi}(x, y, 0+) = e^{-ik(x - x_T)} (x_T, y, 0+)$$

in W. In addition to the set of boxes B on the wing, a corresponding set of boxes B_W on W must be considered. After finding a value $\bar{\psi}_{ij}$ in a box of B along the trailing edge, the formula above may be used to find values in the boxes directly downstream. If the ith row of boxes includes boxes of B_W, to the right side of Equation (33) must be added the contribution of all boxes $B_{i'j'}$ in B_W with $i' \leq i$. The computer program must also be modified in several other respects, to take into account the more general wing shape.

A wing that consists of several almost planar sections in different planes, such as a wing with folded tips, may also be handled by the doublet box method. Equation (33) applies, if $\bar{\Psi}_{ij}$ is interpreted as one-half of the discontinuity in Ψ between the upper and lower surfaces. The influence coefficients involved are given by a more general formula (not given in this report), allowing for out-of-plane influence of the doublets. Formulas analogous to those of Appendix II may be developed, which are not much more complicated. The main effect of this extension on the computer program would be a greater number of distinct values of the influence coefficients, so that it would not be possible to store them all in an array in core unless the limit on the number of boxes in each direction were considerably reduced.

Rectangular boxes, not necessarily square, may be used. Let the boxes have sides of length d_1 chordwise and d_2 spanwise.

If

$$\ell_1 = kd_1, \ \ell_2 = kd_2^2/d_1$$

the formula for the influence coefficients, Equation (39) in Appendix II, must be replaced by

$$A\,(n,\ m) = \frac{ik}{2\pi} \iint\limits_{\substack{|v-m|<1/2 \\ |u-n|<1/2 \\ u>0}} \frac{du\,dv}{u^2}\, e^{-1/2\, i\, (\ell_1 u + \ell_2 v^2/u)}$$

This may be evaluated by the methods of Appendix II. Except for this difference, the method is essentially the same. The best choice of box shape probably depends on the aspect ratio of the wing.

3. DESCRIPTION OF THE COMPUTER PROGRAM

1. COORDINATE SYSTEMS

An initial coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ is assumed, with the $\tilde{x}$-axis in the direction of the flow. The undisturbed position of the wing is in a region S in the $\tilde{x}\tilde{y}$-plane, with the x-axis along the center line and the origin at the nose (see Figure 1). This coordinate system is used in the data.

In the program, a dimensionless coordinate system (x, y, z) is used, based on the root chord length b:

$$x = \tilde{x}/b$$

$$y = \tilde{y}/b$$

$$z = \tilde{z}/b$$

2. WING GEOMETRY

The wing is symmetric, with trailing edge $\tilde{x} = b$. To complete its description, the portion of the leading edge on which $\tilde{y} > 0$ must be specified. This is done by giving the coordinates of the end points of NS line segments along the edge $(1 \leq NS \leq 3)$, beginning at a point at which $\tilde{y} = 0$: $(0, \tilde{y}_o)$, $(\tilde{x}_1, \tilde{y}_1)$, ..., $(\tilde{x}_{NS}, \tilde{y}_{NS})$. The edge of S includes the polygonal line through these points. If $\tilde{y}_o > 0$, it also includes the line from the origin to $(0, \tilde{y}_o)$. If $\tilde{x}_{NS} < b$, it includes the line from $(\tilde{x}_{NS}, \tilde{y}_{NS})$ to $(b, \tilde{y}_{NS})$. (See Figure 3.)

Leading edges of fairly general shape may be approximated by such polygonal lines.

3. THE DEFLECTION DATA

A mode is specified by the vertical deflection function $f(\tilde{x}, \tilde{y})$ in terms of which the equation for the instantaneous position of the planform is

$$\tilde{z} = \mathrm{Re}\,[\delta \cdot e^{i\omega t} f(\tilde{x}, \tilde{y})]$$

where δ is a constant.

In the program, f is assumed to be a polynomial in $\tilde{x}$ and $\tilde{y}^2$. The data may give either the coefficients of this polynomial, or values of f at a

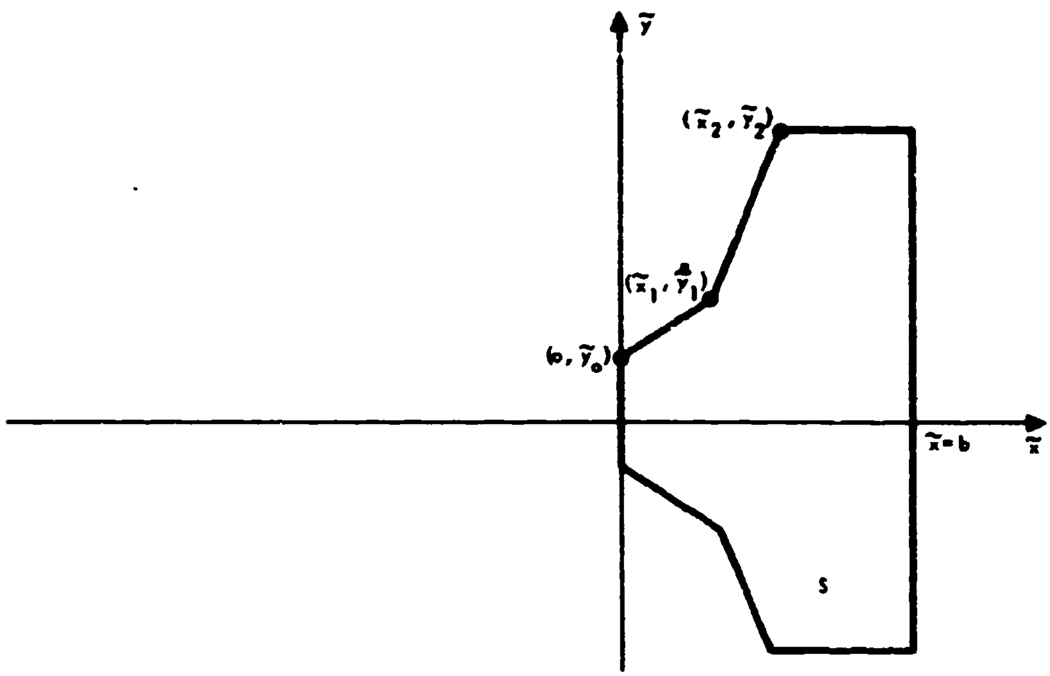


Figure 3. Wing Geometry (NS = 2)

set of points on the wing. In the latter case, a polynomial is fitted to the given values by a least square error technique.

4. LEAST SQUARE SURFACE FITS

The problem involved here is the approximation of a function of x and y by an expression of the form

$$\bar{\varphi}(x, y) = \sum_{n,m} a_{nm}\, x^n\, y^{2m}\, F_{(x,y)}$$

when a set of values of the function is known. This arises in the program in two places. The subroutine DRED fits a representation of the deflection of this type with F = 1 to the given deflection values. In the subroutine BØXP, such a fit is made for the potential, with

$$F(x, y) = \left\{ \begin{array}{l} \sqrt{x^2 - x_o(y)^2} \\ \text{or} \\ \sqrt{x - x_o(y)} \end{array} \right\} \cdot \left\{ \begin{array}{l} \sqrt{1 - y^2/y^2_{max}} \\ \text{or} \\ 1 \end{array} \right\}$$

($x = x_o(y)$ is the equation of the leading edge) depending on the wing shape. This factor approximates the proper behavior of $\bar{\varphi}$ at the edges. The factor $\sqrt{x^2 - x_o(y)^2}$ is used for a pointed nose ($\tilde{y}_o = 0$), and $\sqrt{x - x_o(y)}$ for an unswept nose ($\tilde{y}_o > 0$). The factor $\sqrt{1 - y^2/y^2_{max}}$ is included if the planform has a side edge along which $y = y_{max}$.

The factor $F(x, y)$ is real, so the values of $\bar{\varphi}$ have real and imaginary parts that involve only the corresponding parts of the a_{nm}'s. Hence, these real and imaginary parts may be handled separately, reducing the problem from one in complex numbers to one in real numbers.

Let $\alpha_{nm} = \mathrm{Re}\left[a_{nm}\right]$, and let the real parts of given values of the function at data points be $\bar{\varphi}'_j$ at (x_j, y_j), $j = 1, \ldots, NP$. Then for the real parts we wish to have

$$\sum_{n,m} \alpha_{nm} \, x_j^{\,n} \, y_j^{\,2m} \, F(x_j, y_j) \cong \bar{\varphi}'_j, \quad j = 1, \ldots, NP$$

The least squares method minimizes

$$Q = \sum_j \left[\sum_{n,m} \alpha_{nm} \, x_j^{\,n} \, y_j^{\,2m} \, F(x_j, y_j) - \bar{\varphi}'_j \right]^2$$

(See Reference 11, Chapter 16.)

For condensed notation, let r be a single index over the pairs (n, m), let $\alpha_{nm} = \alpha_r$, and $x_j^{\,n} \, y_j^{\,2m} \, F(x_j, y_j) = A_{jr}$. Then

$$Q = \sum_j \left[\sum_r \alpha_r A_{jr} - \bar{\varphi}'_j \right]^2$$

Let the range of r be from 1 to $NC \leq NP$.

To minimize Q, we set

$$\frac{\partial Q}{\partial \alpha_r} = 0, \quad r = 1, \ldots, NC$$

This leads to the system of equations

$$\sum_{r'} \left(\sum_j A_{jr} A_{jr'} \right) \alpha_{r'} = \sum_j A_{jr} \bar{\varphi}'_j, \quad r = 1, \ldots, NC \tag{34}$$

Put

$$\left.\begin{aligned} \sum_j A_{jr} A_{jr'} &= B_{rr'} \\ \sum_j A_{jr} \bar{\varphi}'_j &= C'_r \end{aligned}\right\} \tag{35}$$

Then Equation (34) reduces to

$$\sum_{r'} B_{rr'} \alpha_{r'} = C'_r, \quad r = 1, \ldots, NC \tag{36}$$

The matrices $(B_{rr'})$ and (C'_r) must be set up to solve Equation (36). It is not necessary, however, to set up the matrix (A_{jr}). Only one row of (A_{jr}) is needed at a time. This is fortunate, because the program allows (A_{jr}) to become as large as 2500 x 20. For each value of j, the jth row of (A_{jr}) is computed, and from this the jth terms in the sums in Equation (35) are formed and added in.

In the complex case, there is a corresponding system of equations for the imaginary parts:

$$\sum_{r'} B_{rr'} \beta_{r'} = C''_r$$

The two systems of equations are solved together by the subroutine XSIMEQ, which allows for more than one set of values on the right.

5. GENERALIZED FORCES

The generalized force coefficient L_{ij} is defined (Reference 6) by

$$L_{ij} = \frac{1}{1/2 \rho U_\infty^2 S} \iint_S \Delta p_i(x, y) f_j(x, y) \, dx \, dy$$

where Δp_i is the lifting pressure difference in the ith mode, and f_j is the deflection function in the jth mode. In terms of the potential $\bar{\varphi}(x, y)$ on the upper surface,

$$\Delta p_i = 2\rho U_\infty^2 (\bar{\varphi}_x + i k \bar{\varphi})$$

$$L_{ij} = \frac{4}{S} \iint_S (\bar{\varphi}_x + i k \bar{\varphi}) f_j \, d x \, d y$$

After integration by parts,

$$L_{ij} = \frac{4}{S} \left\{ \int_{x=1} \bar{\varphi} f_j \, dy + \iint_S \bar{\varphi} \left(i k f_j - \frac{\partial f_j}{\partial x} \right) d x \, d y \right\} \tag{37}$$

In Equation (37) insert the series

$$\bar{\varphi} = \sum_{n, m} a_{nm} x^n y^{2m} F(x, y)$$

$$f_j = \sum_{n', m'} d_{n'm'} x^{n'} y^{2m'}$$

The result is

$$L_{ij} = \frac{8}{S} \sum_{n', m'} d_{n'm'} \sum_{n, m} a_{nm} \left[\frac{1}{2} \int_{x=1} y^{2m+2m'} F(1, y) \, dy \right.$$

$$+ ik \cdot \frac{1}{2} \iint_S x^{n+n'} y^{2m+2m'} F(x, y) \, d x \, d y$$

$$\left. - n' \cdot \frac{1}{2} \iint_S x^{n+n'-1} y^{2m+2m'} F(x, y) \, d x \, d y \right]$$

The integrals in this expression depend only on the wing shape. They are computed by the subroutine FØRCI before the work on the individual modes begins. During the work on the ith mode, the sum over n and m is

performed in the last part of the subroutine BØXP, for each set of values of n' and m'. The sum over n' and m' and multiplication by 8/S is performed in the last part of the main program.

6. THE USE OF GAUSSIAN QUADRATURE IN THE EVALUATION OF GENERALIZED FORCES

Gaussian quadrature is an approximation of the form

$$\int_a^b f(u)\,du \cong \sum_{j=1}^{N} h_j f(u_j)$$

exact for polynomials of degree $\le 2N - 1$. (See Reference 11, Chapter 7.) This formula is used with (a, b) = (0, 1), N = 6. The values of the h_j's and u_j's for this case were obtained from values listed in Reference 11. They are given as H(1),..., H(6), U(1),..., U(6) in the subroutine SECT.

Subroutine FØRCI finds the values of

$$AXY(I,J) = \frac{1}{2} \iint_S x^{I-1} y^{2J-2} F(x,y)\,dx\,dy$$

and

$$AY(J) = \frac{1}{2} \int_{x=1} y^{2J-2} F(1,y)\,dy$$

for I, J = 1,..., 9. To do this, the contributions to the integrals from each section of wing behind a straight piece of leading edge are calculated separately in SECT.

The form of F(x, y) is

$$F(x,y) = \left\{ \begin{array}{c} \sqrt{x - x_o(y)} \\ \text{or} \\ \sqrt{x^2 - x_o(y)^2} \end{array} \right\} \cdot \left\{ \begin{array}{c} \sqrt{1 - y^2/y^2_{max}} \\ \text{or} \\ 1 \end{array} \right\}$$

depending on the wing shape. We have integrals that behave like square roots at the leading edge. The integrals over one wing section are of the form

$$BXY = \int_{y-}^{y+} dy \int_{x_o(y)}^{1} dx\, x^{I-1}\, y^{2J-2}\, F(x,y)$$

and

$$BY = \int_{y_-}^{y_+} dy\, y^{2J-2}\, F(1,y)$$

In BXY, the chordwise integral is evaluated first at each value of y at which it will be needed. The new variable

$$u = \sqrt{x - x_o(y)} \Big/ \sqrt{1 - x_o(y)} \qquad (38)$$

is introduced. Then

$$\int_{x_o(y)}^{1} dx\, x^{I-1}\, y^{2J-2}\, F(x,y) = \int_{0}^{1} du \cdot 2\left[1 - x_o(y)\right] x^{I-1}\, y^{2J-2}\, F(x,y)$$

The integrand, as a function of u, is well-behaved at the leading edge. It is approximated by

$$\sigma(y) = \sum_{i=1}^{6} h_i \cdot 2\left[1 - x_o(y)\right] x_i^{\,I-1}\, y^{2J}\, F(x_i, y)$$

where x_i is computed from the value of u_i according to Equation (38).

In the y-integration in BXY and BY, the integrand approaches zero as as $y \to y_{max}$ like $\sqrt{1 - y/y_{max}}$ or $(1 - y/y_{max})^{3/2}$. Accordingly, the change of variable

$$y = \begin{cases} y_+ - (y_+ - y_-)v, & y_+ < y_{max} \\ y_+ - (y_+ - y_-)v^2, & y_+ = y_{max} \end{cases}$$

is used, which makes the interval of integration $0 < v < 1$ and removes the square root behavior in the last section of the wing. This leads to the formulas

$$BXY = (y_+ - y_-) \sum_{j=1}^{6} h_j \, \sigma(y_j) \cdot \begin{cases} 1, & y_+ < y_{max} \\ 2u_j, & y_+ = y_{max} \end{cases}$$

$$BY = (y_+ - y_-) \sum_{j=1}^{6} h_j \, y_j^{2J-2} \, F(1, y_j) \cdot \begin{cases} 1, & y_+ < y_{max} \\ 2u_j, & y_+ = y_{max} \end{cases}$$

7. LEADING EDGE CORRECTION

The value of potential found for each box from Equation (33) is taken to be the value of $\bar{\varphi}$ at the box center. Thus, the values obtained are in error only by virtue of the error introduced in the values of upwash when the actual distribution of potential in a box is replaced by this constant value. This error is especially important in the first row of boxes, for a wing with an unswept leading edge. The major effect is on the upwash values in that row.

To estimate this error, consider the two-dimensional case, in which $\bar{\varphi}$ is independent of y. In Equation (26), the expression for upwash due to a doublet distribution, integrate by parts over ξ, then integrate over η. The result is

$$\bar{w}(x,y) = \frac{ik}{2\pi} \iint_{0<\xi<x} \frac{d\xi\, d\eta}{(x-\xi)^2} \bar{\varphi}(\xi, \eta, 0+)\, e^{-\frac{1}{2} ik \left(x - \xi + \frac{(y-\eta)^2}{x-\xi}\right)}$$

$$= \frac{1}{\pi} \iint_{0<\xi<x} \frac{d\xi\, d\eta}{(y-\eta)^2} \left(\bar{\varphi}_\xi + \frac{1}{2} ik\bar{\varphi}\right) e^{-\frac{1}{2} ik \left(x - \xi + \frac{(y-\eta)^2}{x-\xi}\right)}$$

$$= -\sqrt{\frac{2ik}{\pi}} \int_0^x \frac{d\xi}{\sqrt{x-\xi}}\, e^{-\frac{1}{2} ik(x-\xi)} \left(\bar{\varphi}_\xi + \frac{1}{2} ik\bar{\varphi}\right)$$

For $x = \frac{1}{2} d$, if kd is small,

$$\bar{w}\left(\frac{1}{2}d, y\right) \cong -\sqrt{\frac{2ik}{\pi}} \int_0^{\frac{1}{2}d} \frac{d\xi}{\sqrt{\frac{1}{2}d - \xi}} \bar{\varphi}_\xi(\xi, \eta, 0+)$$

The correct leading edge behavior is possessed by the expression $\bar{\varphi} = C\sqrt{\xi}$. We have

$$\bar{w}\left(\frac{1}{2}d, y\right)\Big|_{\bar{\varphi} = C\sqrt{\xi}} = -\sqrt{\frac{2ik}{\pi}} \frac{C}{2} \int_0^{\frac{1}{2}d} \frac{d\xi}{\sqrt{\xi\left(\frac{1}{2}d - \xi\right)}} = -\sqrt{\frac{2ik}{\pi}} C \frac{\pi}{2}.$$

If $\bar{\varphi}$ is constant on $0 < \xi < d$, and has the value $C\sqrt{1/2\, d}$, then $\bar{\varphi}_\xi$, in the above integral, can be expressed in terms of a delta function:

$$\bar{\varphi}_\xi = C\sqrt{\frac{1}{2}d}\ \delta(\xi)$$

Accordingly,

$$\bar{w}\left(\frac{1}{2}d, y\right)\Big|_{\bar{\varphi} = C\sqrt{\frac{1}{2}d}} = -\sqrt{\frac{2ik}{\pi}} C.$$

Note that the latter value is smaller than the value of $\bar{w}$ evaluated for $\bar{\varphi} = C\sqrt{\xi}$ by the factor $2/\pi$. This implies that the values of potential found for the first row of boxes will be more accurate if the upwashes in that row are multiplied by $2/\pi$.

8. THE FORM OF OUTPUT

The viewpoint is taken that calculation of the generalized forces is the basic purpose of the program. They are always printed out. There are other outputs that will be printed if the appropriate data signal is given. Each of the following is printed if the data item specified in parentheses is non-zero:

a. The coefficients of the deflection polynomial, if it has been computed as a fit to given values of deflection, DA(87)

b. The upwash array, DA(88)

c. The potential array, DA(89)

d. The coefficients of the potential series, DA(90)

e. Values of pressure and potential at the box centers, computed from the series, DA(91).

9. THE DATA SUBROUTINE DATRD

This subroutine reads all data items into the array DA. Punched cards used for data are considered to contain six fields of length 12 as indicated in the sample data sheets. The first field contains information for DATRD. Ending in column 12 is an integer giving the location in the data array for the entry in the second field. The following fields go into consecutive locations, if the data are numeric. Floating point numbers should be written with decimal points, and fixed point numbers adjusted to the right end of the field.

The word ALPHA in columns 2 through 6 indicates that the data on the card are alphanumeric. These are stored in DA in a different way, taking up ten locations per card. The data may be printed later, just as they appeared on the card.

On a numeric card, if a field is blank the corresponding location in DA is unchanged. This is not true for an ALPHA card.

A minus sign in column 1 indicates the last card to be read at the time. DATRD reads cards until this minus sign is encountered, then returns to the main program.

10. A NOTE ON THE USE OF TAPES

In writing of this program, the following tape numbers have been used: output tape, number 6; input tape, number 5; and tape simulated by an internal file, number 99.

The tape numbers 5 and 99 appear in the subroutine DATRD. Elsewhere only the output tape number is used. It occurs in the main program, and in the subroutines SHAPE, DRED, BØXP and BØXPØ.

11. USE OF THE PROGRAM FOR FIXED WING AND MODES AT VARIOUS FREQUENCIES

If a non-zero quantity is entered in the appropriate location in the data array, (DA26), it indicates that a wing shape and set of modes to be used are the same already used for another frequency. Then quantities that depend only on wing shape and deflection data will not be computed, but will be taken from the permanent arrays in which they were stored in the previous case. The number of boxes along the root chord, DA(27), may not be changed when this is done.

When this option is exercised, all work for the present frequency will be carried out after reading one set of data, which need only include the frequency and the indicator, DA (26). Titles for the individual modes are not printed.

12. DESCRIPTION OF THE DATA ARRAY

All data are entered into the array DA, dimensioned for 700, as described in Paragraph 9. The layout of the array is as follows:

1 - 10	Title
13 - 22	Mode Title
23 :	Frequency (cycles per unit of time), ν
24 :	Root chord length, b
25 :	Speed of sound, a
26 :	Indicator for new frequency (See Paragraph 11)
27 :	Number of boxes along root chord
28 :	Number of modes
29 :	Number of sections of leading edge to be given (See Paragraph 2)
30 - 36	Coordinates of points on the leading edge (See Paragraph 2)
39 :	Indicator to suppress calculation of potential for a mode
46 - 70	Coefficients of the deflection polynomial (See Paragraph 3)
87 - 91	Output indicators (See Paragraph 8)
98 :	Number of points at which deflections are given (See Paragraph 3)
99 :	Number of $\tilde{x}$ values
100 :	Number of $\tilde{y}$ values
101 - 700	Deflection data for a maximum of 150 points

Note: 23, 24. 25 and 30-36 must be entered in consistent units of length and time.

13. OUTLINE OF THE PROGRAM

For the purpose of description, the main program has been divided into 20 parts, as indicated in Figure 4, which shows the flow of the program and the subroutines called.

14. SIZE LIMITATIONS OF THE PROGRAM

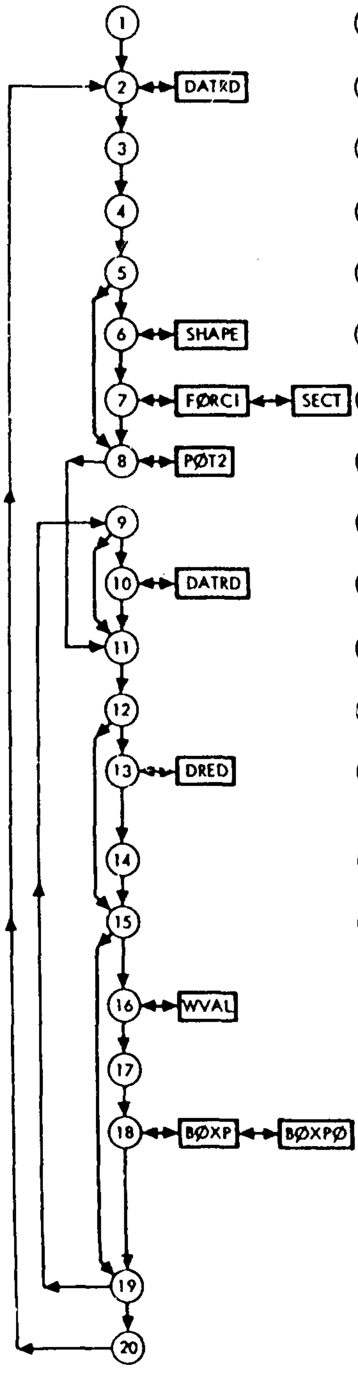
The program's size limitations are as follows:

a. Box size – the half wing must be enclosed in a rectangle that contains no more than 50 boxes in each direction. (The use of a large number of boxes is not recommended, because the time required is roughly proportional to the cube of the number of boxes along the root chord. The possibility of 50 boxes in each direction is intended to allow a large range in aspect ratio.)

b. Number of modes – ten at most.

c. Number of points at which the deflections are given for one mode – 150 at most.

d. Terms in the deflection polynomial – this is $\sum d_{nm} x^n y^{2m}$, where $0 \le n \le 4$, $0 \le m \le 4$.



(1) The data array is initialized by making the titles blank and weights 1.0.

(2) Subroutine DATRD reads the first block of data

(3) The following quantities are computed: reduced frequency $k = 2\pi\nu b/a$, D = dimensionless box size, DH = 0.5D.

(4) The title is printed, with the values of L (the number of boxes along the chord), k, and ν.

(5) If DA(26) ≠ 0 (See Paragraph 11 of this section) control transfers to Part (8)

(6) Subroutine SHAPE computes the number of boxes in each row and the wing area.

(7) Subroutine FØRCI computes the integrals to be used in evaluation of generalized forces (See Paragraphs 5, 6 of this section).

(8) Subroutine PØT2 computes the influence coefficients. Control goes to Part (11)

(9) If DA(26) ≠ 0 (see Paragraph 11 of this section) control goes to Part (11)

(10) DATRD reads the next block of data.

(11) The mode number and title are printed.

(12) If DA(26) ≠ 0, control goes to Part (15)

(13) The subroutine DRED computes the deflection polynominal if deflection values were given. The coefficients are transformed into coefficients of a polynominal in the dimensionless variables x, y and stored in the permanent array DF.

(14) G(M) = DA(39)

(15) If G(M) ≠ 0, this indicates that the potential should not be computed for the current mode. In this case, this mode will be used only as a deflection in the computation of generalized forces. Control goes to Part (19)

(16) Subroutine WVAL sets up the array of downwash values W.

(17) The first row of values in W are corrected as described in Paragraph 7 of this section.

(18) Subroutine BØXP is called. The subroutine BØXPØ solves Equation (33) to get the array of potential values. A least-squares fit is made to these values as described in Paragraph 8 of this section. The print options (b), (c), (d), (e) listed in Paragraph 8 of this section are handled by this subroutine. The final section of the subroutine stores in the array PS the sums over the potential coefficients a_{nm} shown in Paragraph 5 of this section.

(19) If there are more modes, control returns to Part (9)

(20) The generalized forces are computed by summing over the deflection coefficients d_{nm} as described in Paragraph 5 of this section. Control goes to Part (2)

Figure 4. Program Flow Diagram

4. RESULTS

1. THE ASPECT RATIO 1.5 DELTA WING

The computer program was run for the plunging and pitching modes (pitch axis at $x = 0$) at the reduced frequencies $k = 0.2, 0.5, 0.8, 1.0$. Forty boxes along the root chord were used, which leads to about 300 boxes on the half-wing.

Theoretical values for comparison were calculated from Davies' formulas (Reference 12). These are analytic expressions of the solution of Equation (5) for the potential and generalized forces for the delta wing in rigid modes of oscillation, expressed as series in k. Figures 5 through 7 show the values of generalized forces L_{11} (lift due to plunge), L_{21} (lift due to pitch), and L_{22} (moment due to pitch). Note that the vertical scales have been expanded in the portion of interest, especially for L_{11}. Most of the values agree to within 2 or 3 percent.

The differences indicate the errors introduced by the box method in the solution of Equation (5), as distinguished from the errors inherent in this equation.

Figure 8 gives the chordwise distribution of values of $\bar{\Phi}$ for the plunging mode at $k = 0.5$, for $y = y_{max}/3 = 0.125$.

2. THE ASPECT RATIO 2.0 RECTANGULAR WING

The plunging and pitching modes were again used at $k = 0.3, 0.6, 0.9$. Twenty-five boxes were allowed along the chord, giving 625 boxes on the half-wing. The values of L_{21} and L_{22} are shown in Figures 9 and 10, with values from Landahl (Reference 6, page 84) for comparison. Landahl's values were obtained by a method of solution of Equation (5) which applies only to a rectangular wing in modes of oscillation with a deflection independent of y.

3. THE ASPECT RATIO 3.0 RECTANGULAR WING

Finally, for the aspect ratio 3.0 rectangular wing, a comparison is made with experimental pressure values. These values were given in Reference 13 for a 5-percent thickness wing oscillating in an elastic bending mode. At Mach 1, the reduced frequency was 0.24. The chordwise pressure distribution at $y = y_{max}/2$ is shown in Figure 11.

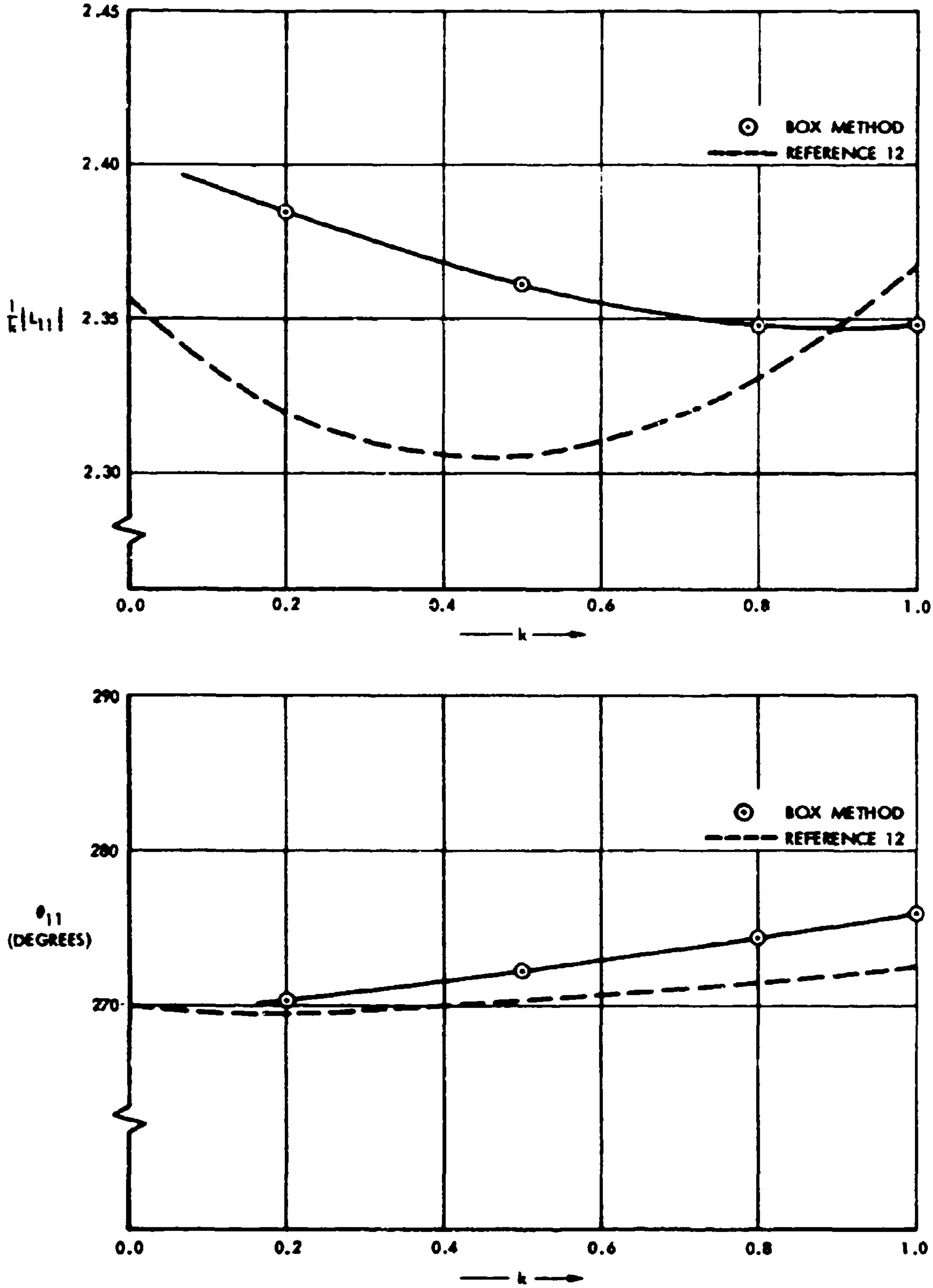


Figure 5. Lift Due to Translation for an Aspect Ratio 1.5 Delta Win (Compared With Reference 12)

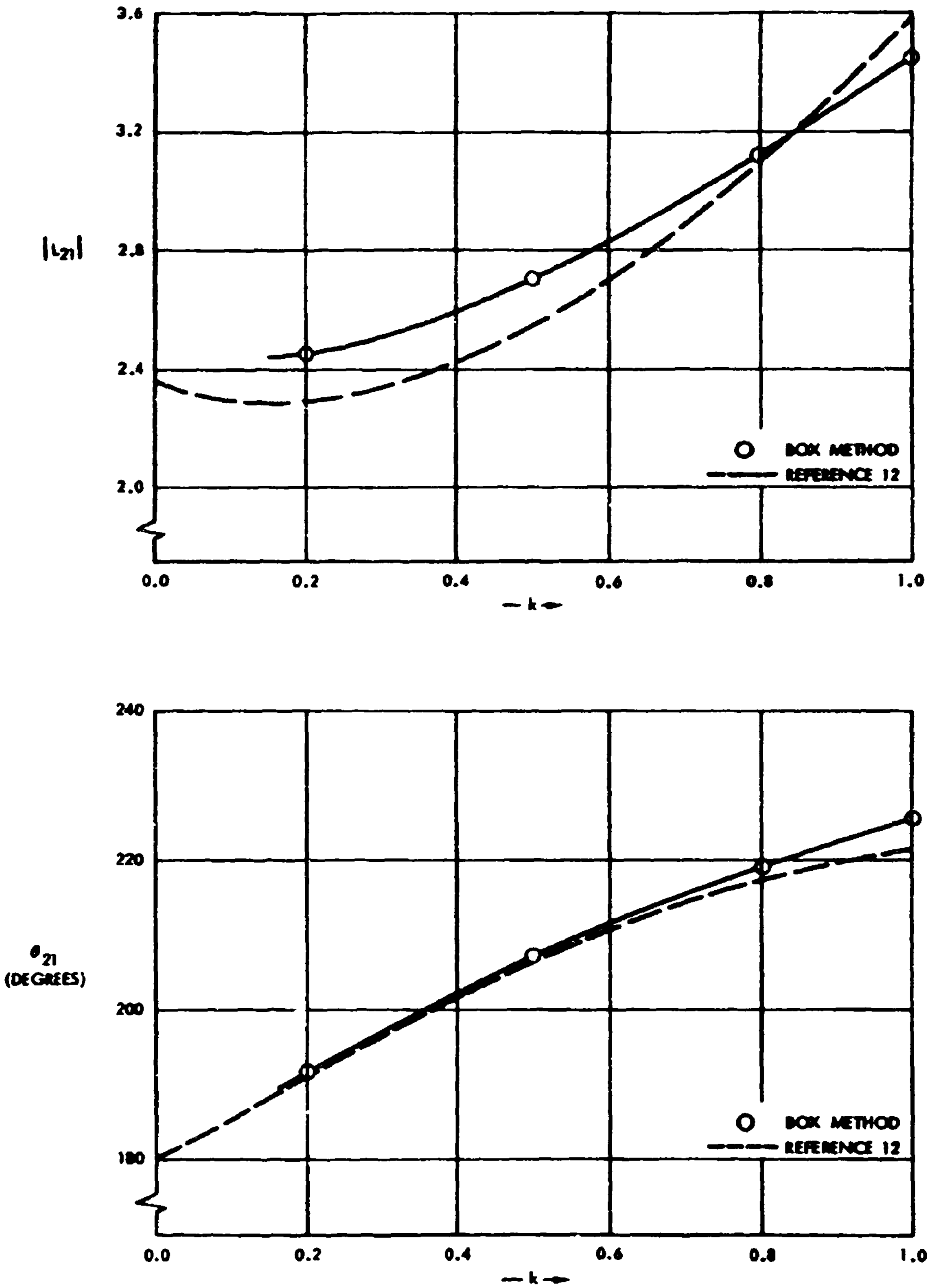


Figure 6. Lift Due to Pitch for an Aspect Ratio 1.5 Delta Wing (Compared With Reference 12)

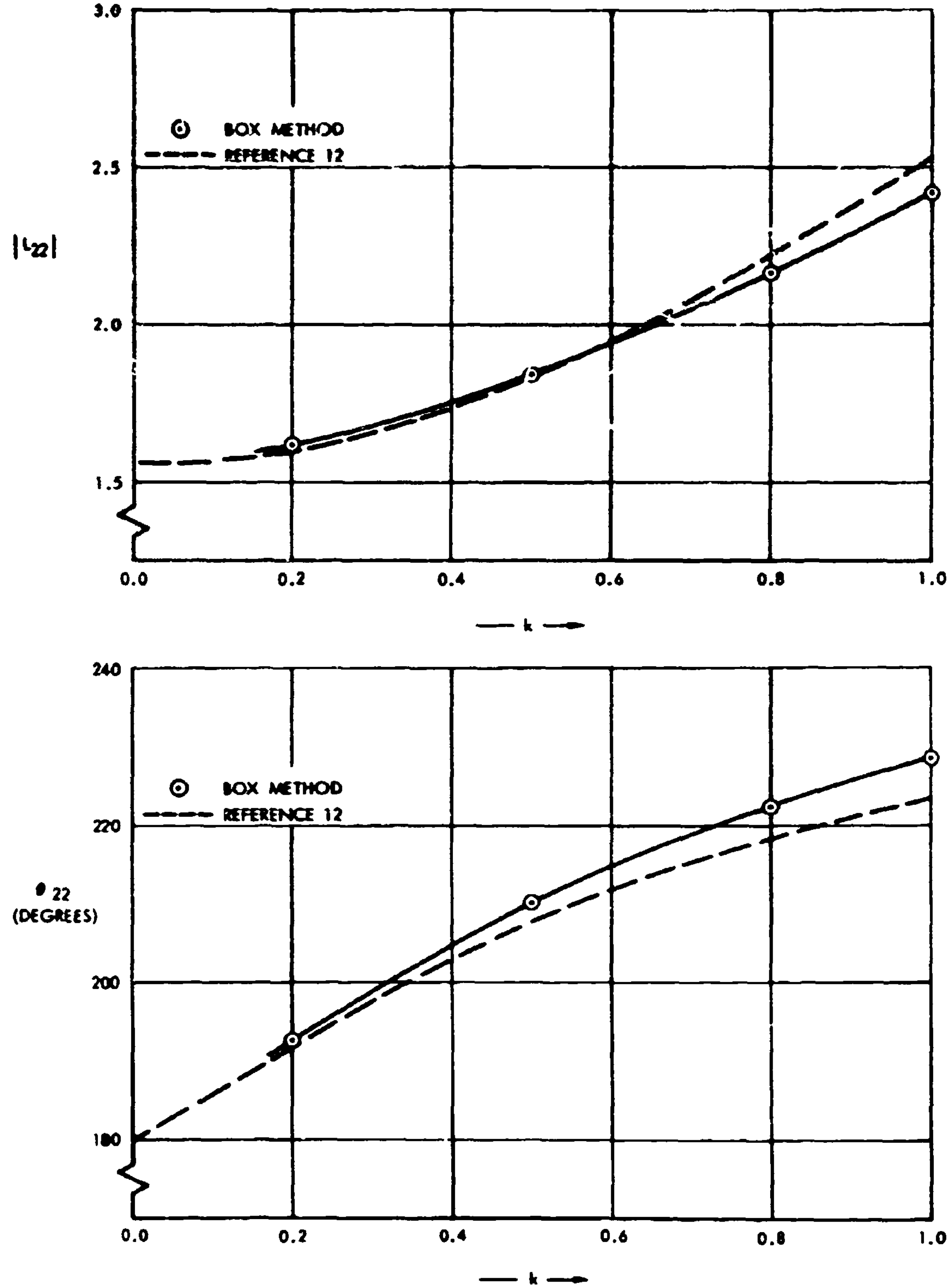


Figure 7. Moment Due to Pitch for an Aspect Ratio 1.5 Delta Wing (Compared With Reference 12)

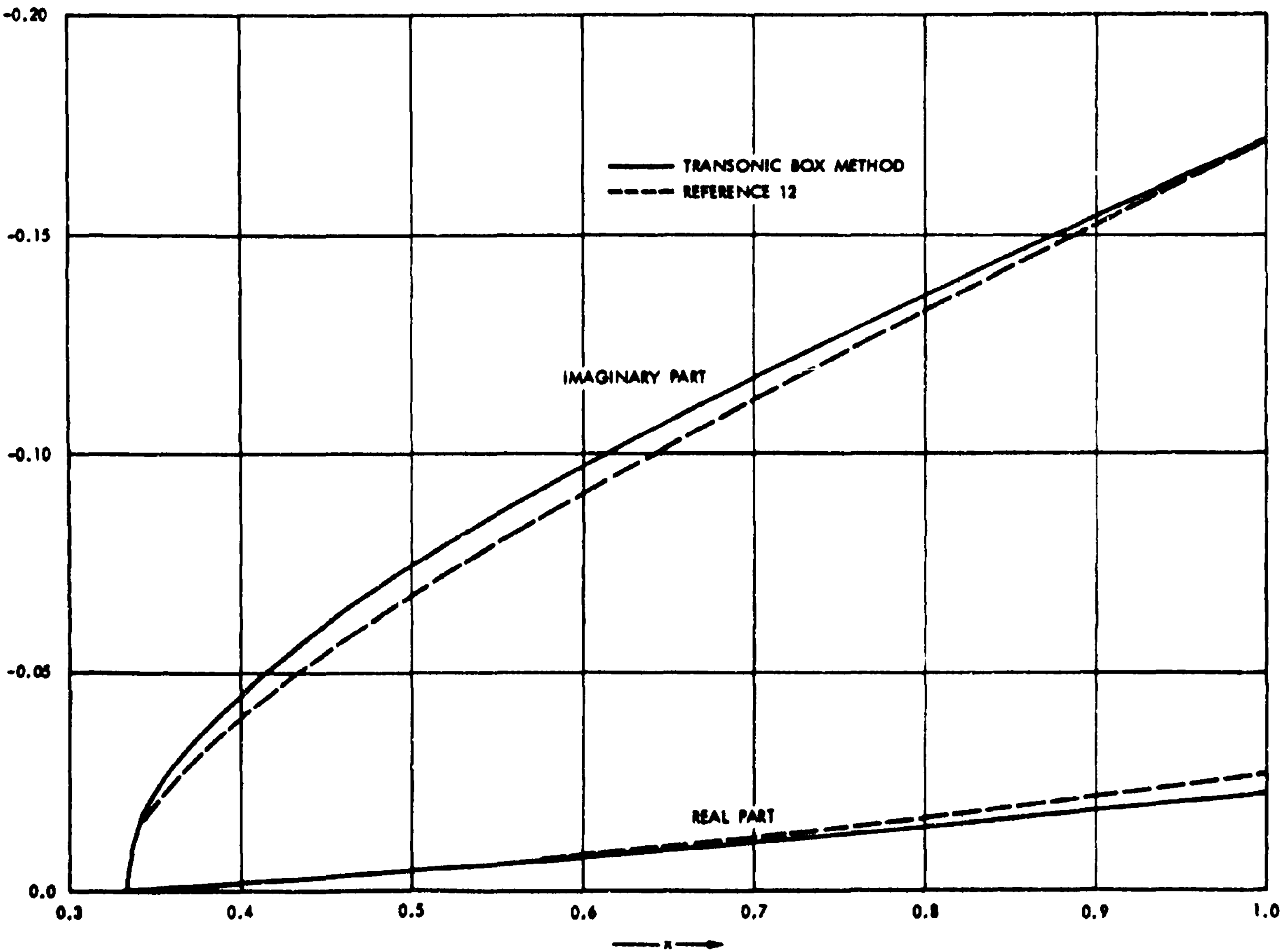


Figure 8. Real and Imaginary Parts of the Unsteady Potential $\bar{\varphi}$ in the Plunging Mode for an Aspect Ratio 1.5 Delta Wing at $k = 0.5$; Chordwise Distribution for $y = 0.125 = g$ Max/3 (Compared With Reference 12)

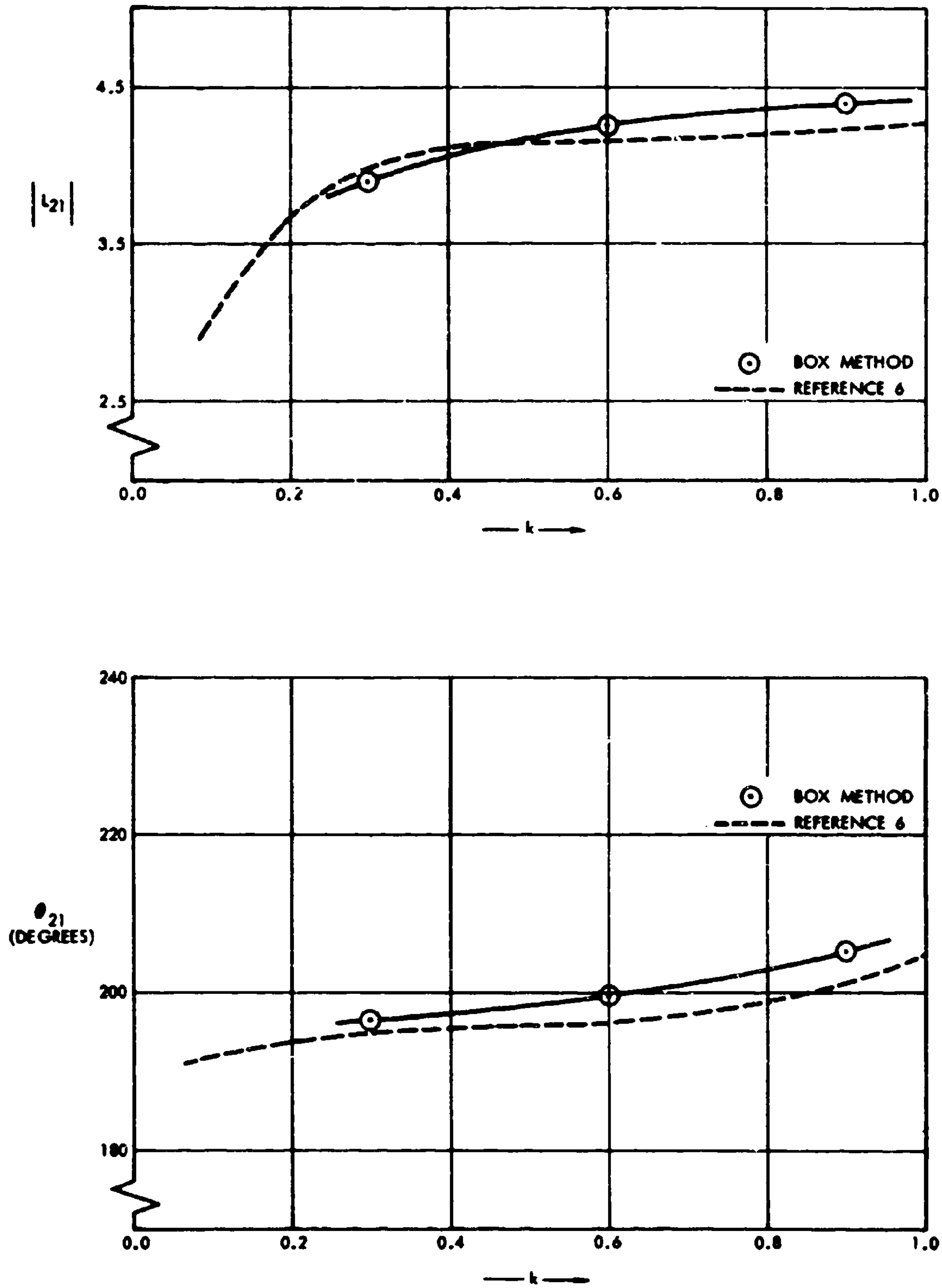


Figure 9. Lift Due to Pitch for an Aspect Ratio 2.0 Rectangular Wing (Compared With Reference 6)

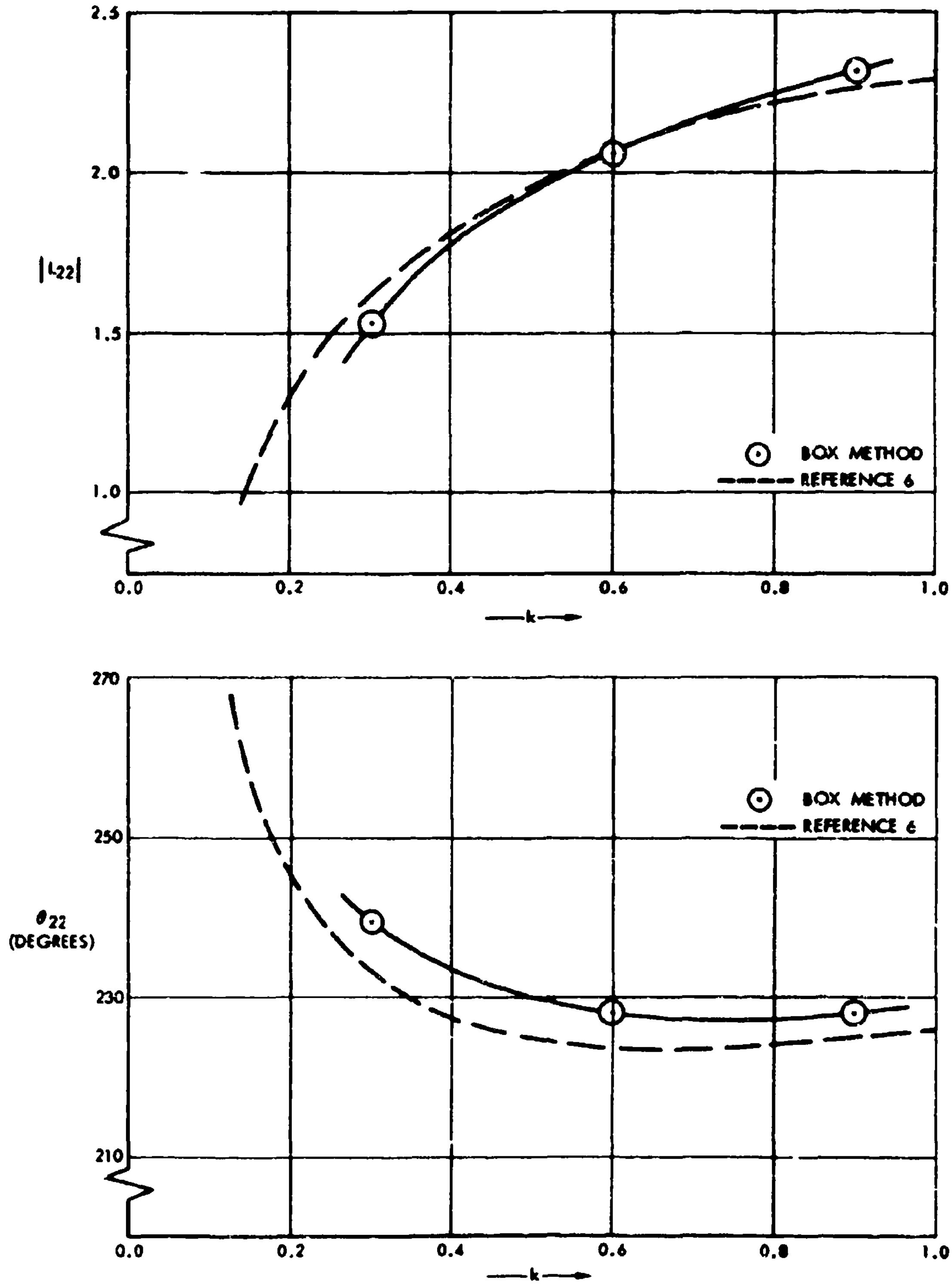


Figure 10. Moment Due to Pitch for an Aspect Ratio 2.0 Rectangular Wing (Compared With Reference 6)

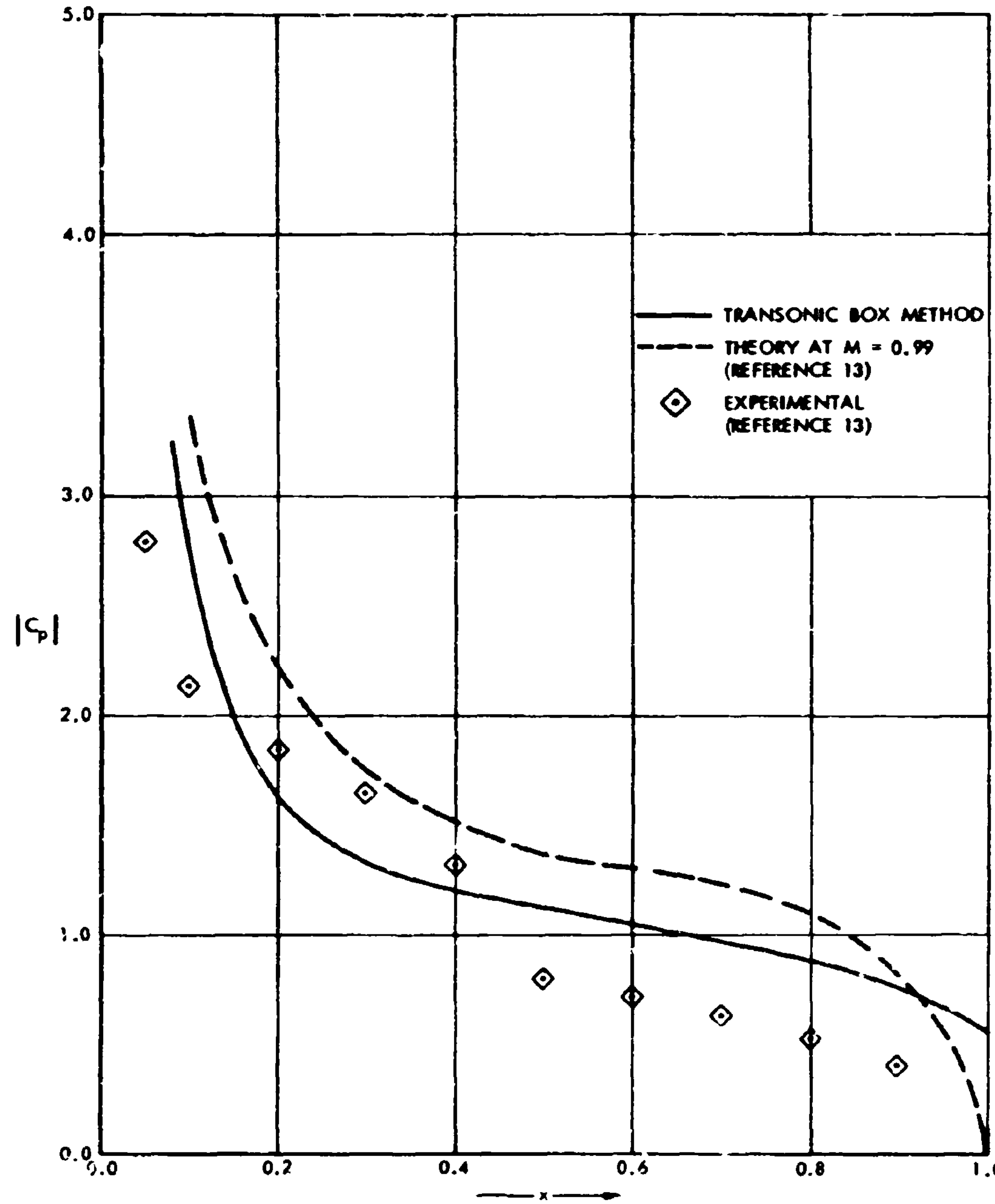


Figure 1[illegible]. Chordwise Pressure Distribution on an Aspect Ratio 3.0 Rectangular Wing in an Elastic Mode; $y = y_{max}/2 = 0.75$ (Compared With Reference 13)

The variation of the experimental values from the computed values is of the type that thickness effects should be expected to cause: the measured pressure is (1) smaller near the leading edge, (2) larger before the point of maximum thickness ($x = 0.5$), and (3) smaller beyond this point. The experimentally determined values of local Mach number along this chord range from 0.84 to 1.35, which indicates that the thickness has a considerable effect on the flow.

The theoretical curve given in Reference 13 was obtained from the subsonic kernel function method, applied at $M = 0.99$. This curve is included to show how another theoretical method compares with the experimental values.

4. COMPUTER RUNNING TIME

The results described in this section required about 20 minutes total computer time on the IBM 7094. With nonessential output omitted, this time could have been reduced. All optional output was given, resulting in about 40,000 lines of output.

5. CONCLUSIONS

A procedure has been developed for predicting unsteady aerodynamic forces and pressures on an oscillating wing by the use of the transonic box method. The results obtained by this method agree quite well with theoretical values from other methods that are applicable only to special planforms. The box method has the advantage of applicability to a general planform. The only other method of this generality at Mach 1 is the sonic limit of the subsonic kernel function method (see Reference 16) that has not been very successful.

The comparison with experimental values in Figure 11 indicates that the most serious limitation of the method is that thickness is neglected. Thickness may be incorporated into a box program by using modified forms for sources and doublets, depending on the local Mach number (see Reference 14). This was not accomplished under the present program. Other possible extensions of the transonic box method, that would not require much change in the existing computer program, are described in Paragraph 10 of Section 2.

REFERENCES

1. Stark, V. J. E. Calculation of Aerodynamic Forces on Two Oscillating Finite Wings at Low-Supersonic Mach Numbers. SAAB Tech. Note 53 (Feb. 1964).

2. Zartarian, G., and P. T. Hsu. Theoretical Studies on the Prediction of Unsteady Supersonic Airloads on Elastic Wings, Parts I and II. Wright Air Development Center Technical Report 56-97 (1955).

3. Pines, S., J. Dugundji, and J. Neuringer. "Aerodynamic Flutter Derivatives for a Flexible Wing with Supersonic and Subsonic Edges," Journal of the Aeronautical Sciences, Vol. 22, No. 10 (Oct. 1955).

4. Li, T. C. Aerodynamic Influence Coefficients for an Oscillating Finite Thin Wing, Part I. Chance Vought Aircraft, Inc. Report (June 1954).

5. Bisplinghoff, R. L., H. Ashley, and R. L. Halfman. Aeroelasticity. Cambridge: Addison-Wesley (1955).

6. Landahl, M. T. Unsteady Transonic Flow. New York: Pergamon Press (1961).

7. Lomax, H., M. A. Heaslet, F. B. Fuller, and L. Sluder. Two- and Three-Dimensional Unsteady Lift Problems In High-Speed Flight. NACA Report 1077 (1952).

8. Heaslet, M. A., and H. Lomax. The Use of Source-Sink and Doublet Distributions Extended to the Solution of Boundary Value Problems in Supersonic Flow. NACA Report 900 (1948).

9. Ashley, H., S. Widnall, and M. T. Landahl. New Directions in Lifting Surface Theory. AIAA Preprint 64-1 (January 1964).

10. Evvard, J. C. Use of Source Distributions for Evaluating Theoretical Aerodynamics of Thin Finite Wings at Supersonic Speeds. NACA Report 951 (1950).

11. Scarborough, J. B. Numerical Mathematical Analysis. Baltimore: John Hopkins Press, Fourth Edition (1958).

12. Davies, D. E. "Three Dimensional Sonic Theory," AGARD Manual on Aeroelasticity, Part II, Chapter 4, pp. 74-76.

13. Lessing, H. C., J. L. Troutman, and G. P. Menees. Experimental Determination of the Pressure Distribution on a Rectangular Wing Oscillating in the First Bending Mode for Mach Numbers from 0.24 to 1.30. NASA Technical Note D-344 (Dec. 1960).

14. Landahl, M. T. "Linearized Theory for Unsteady Transonic Flow," Symposium Transsonicum, presented at the IUTAM Symposium, Aachen, Germany (1962), pp. 414 - 439.

15. Pierce, B. O. A Short Table of Integrals. New York: Ginn and Company, Fourth Edition (1956).

16. Runyan, H. L. and Woolston, D. S. Method for Calculating the Aerodynamic Loading on an Oscillating Finite Wing in Subsonic and Sonic Flow. NACA Technical Note 3694 (August 1956).

APPENDIX I. PROPERTIES OF SOURCE AND DOUBLET DISTRIBUTIONS

1. BOUNDARY BEHAVIOR OF A DOUBLET DISTRIBUTION

We wish to evaluate

$$\bar{\varphi}_d(x, y, 0+) = \lim_{z \to 0+} \iint_{\xi > 0} d\xi \, d\eta \, \rho(\xi, \eta) \, \bar{\varphi}_1(x-\xi, \; y-\eta, \; z) .$$

The integrand is zero for $\xi > x$. If we define $\rho(\xi, \eta) = 0$ for $\xi < 0$, then

$$\bar{\varphi}_d(x, y, 0+) = \lim_{z \to 0+} \iint_{\xi < x} d\xi \, d\eta \, \rho(\xi, \eta) \, \bar{\varphi}_1(x-\xi, \; y-\eta, \; z) .$$

Put

$$\xi = x - z^2 u$$

$$\eta = y - zv$$

Then, using Equation (14), we have

$$\bar{\varphi}_d(x, y, 0+) = \lim_{z \to 0+} \iint_{u > 0} du dv \; \rho(x-z^2u, \; y-zv) \cdot z^3 \, \bar{\varphi}_1(z^2u, \; zv, \; z)$$

$$= \lim_{z \to 0+} \iint_{u>0} du dv \; \rho(x-z^2u, y-zv) \cdot \frac{ik}{2\pi} \frac{1}{u^2} \, e^{-\frac{1}{2} ik\left(z^2u + \frac{1+v^2}{u}\right)}$$

Let (x, y) be a point at which ρ is continuous. Then the value of ρ in the integrand approaches $\rho(x, y)$ as $z \to 0$. Taking the limit under the integral sign,

$$\bar{\varphi}_d(x, y, 0+) = \rho(x, y) \cdot \frac{ik}{2\pi} \iint_{u>0} du dv \cdot \frac{1}{u^2} \, e^{-\frac{1}{2} ik \frac{1+v^2}{u}}$$

Let $v = s\sqrt{u}$. Then

$$\overline{\varphi}_d(x, y, 0+) = \rho(x, y) \cdot \frac{ik}{2\pi} \int_0^{\infty} \frac{du}{u^{3/2}} e^{-\frac{1}{2} ik/u} \int_{-\infty}^{\infty} ds\, e^{-\frac{1}{2} iks^2}$$

These integrals may be reduced to a standard form by rotating the paths integration in the complex plane into positions in which the exponents are negative, then making the substitutions

$$u = \frac{ik}{2p}$$

$$s = \sqrt{\frac{2}{ik}}\, q$$

The result is

$$\overline{\varphi}_d(x, y, 0+) = \rho(x, y) \cdot \frac{1}{\pi} \int_0^{\infty} e^{-p} p^{-\frac{1}{2}} dp \int_{-\infty}^{\infty} e^{-q^2} dq$$

$$= \rho(x, y)$$

since both integrals have the value $\sqrt{\pi}$ (see Reference 12, formulas 507, 512). Consequently, Equation (18) is valid at any point of continuity of $\rho(x, y)$.

2. BOUNDARY BEHAVIOR OF A SOURCE DISTRIBUTION

To evaluate $\overline{\varphi}_{sz}(x, y, 0+)$, note that

$$\frac{\partial \overline{\varphi}_s}{\partial z} = \iint_{\xi > C} d\xi d\eta\ \rho(\xi, n) \frac{\partial}{\partial z} \overline{\varphi}_o(x-\xi,\ y-\eta,\ z)$$

$$= \iint_{\xi > C} d\xi d\eta\ \rho(\xi, \eta) \overline{\varphi}_1(x-\xi,\ y-\eta,\ z)$$

$$= \overline{\varphi}_d$$

Hence, by the result of the preceding section, if (x, y) is a point at which $\rho(x, y)$ is continuous,

$$\bar{\varphi}_{sz}(x, y, 0+) = \bar{\varphi}_d(x, y, 0+) = \rho(x, y)$$

which verifies Equation (17).

APPENDIX II. EXPRESSIONS FOR THE INFLUENCE COEFFICIENTS

Equation (31) may be expressed more conveniently in terms of

$$u = (x_i - \xi)/d$$

$$v = (y_j - \eta)/d$$

$$m = |j - j'|$$

$$n = i - i'$$

$$\ell = kd$$

(d = box side length). By Equation (14) we have

$$A(n, m) = \frac{ik}{2\pi} \iint\limits_{\substack{|v-m|<\frac{1}{2} \\ |u-n|<\frac{1}{2} \\ u \geq 0}} \frac{du\,dv}{u^2}\, e^{-\frac{1}{2} i\ell \left(u + \frac{v^2}{u}\right)} \tag{39}$$

It is assumed that ℓ is small. If $\ell < 0.1$, the following approximation gives an error of less than 0.1 percent in the value of A:

$$e^{-\frac{1}{2} i\ell u} = e^{-\frac{1}{2} i\ell n}\, e^{-\frac{1}{2} i\ell (u - n)}$$

$$\cong e^{-\frac{1}{2} i\ell n} \left[1 - \frac{1}{2} i\ell (u - n) - \frac{1}{8} \ell^2 (u - n)^2\right]$$

This reduces Equation (39) to

$$A(n,m) = \frac{ik}{2\pi} e^{-\frac{1}{2} i\ell n} \Bigg\{ \left(1 + \frac{1}{2} i\ell n - \frac{1}{8} \ell^2 n^2\right) \iint \frac{du\, dv}{u^2} e^{-\frac{1}{2} i\ell\, v^2/u}$$

$$+ \left(-\frac{1}{2} i\ell + \frac{1}{4} \ell^2 n\right) \iint \frac{du\, dv}{u} \; e^{\frac{1}{2} i\ell\, v^2/u} \tag{40}$$

$$- \frac{1}{8} \ell^2 \iint du\, dv\, e^{-\frac{1}{2} i\ell\, v^2/u} \Bigg\}$$

where the limits of integration are the same as in Equation (39).

The following formula expresses these double integrals in terms of single integrals:

$$\int_{u_1}^{u_2} du \int_{v_1}^{v_2} dv \frac{1}{u^p} e^{-\frac{1}{2} i\ell\, v^2/u} = \frac{1}{3 - 2p} \int_{u_1}^{u_2} \frac{v du}{u^p} e^{-\frac{1}{2} i\ell v^2/u} \Bigg|_{v = v_1}^{v_2}$$

$$+ \frac{2}{3 - 2p} \int_{v_i}^{v_2} \frac{dv}{u^{p-1}} e^{-\frac{1}{2} i\ell v^2/u} \Bigg|_{u = u_1}^{u_2}$$

Equation (40) becomes

$$A(n,m) = \frac{ik}{2\pi} e^{-\frac{1}{2} i\ell n} \cdot \begin{cases} A_n(n + \frac{1}{2}, m) - A_n(n - \frac{1}{2}, m), & n > 0 \\ A_o(\frac{1}{2}, m), \quad n = 0 \end{cases} \tag{41}$$

where

$$
\begin{aligned}
A_n(u,m) \;=\; v \int_0^u du\, e^{-\frac{1}{2} i\ell v^2/u} \Bigg[& -\frac{1}{u^2}\left(1+\frac{1}{2} i\ell n - \frac{1}{8}\ell^2 n^2\right) \\
& + \frac{1}{u}\left(-\frac{1}{2} i\ell + \frac{1}{4}\ell^2 n\right) - \frac{1}{24}\ell^2 \Bigg] \Bigg|_{v\,=\,m-\frac{1}{2}}^{m+\frac{1}{2}} \\
& + \Bigg[-\frac{2}{u}\left(1+\frac{1}{2} i\ell n - \frac{1}{8}\ell^2 n^2\right) + 2\left(-\frac{1}{2} i\ell + \frac{1}{4}\ell^2 n\right) \qquad (42) \\
& - \frac{1}{12}\ell^2 u \Bigg] \int_{m-\frac{1}{2}}^{m+\frac{1}{2}} dv\, e^{-\frac{1}{2} i\ell v^2/u} \\
= \; & B_n(u,m) + C_n(u,m)
\end{aligned}
$$

B_n and C_n denote the contributions of the terms containing the u - integrals and v - integrals, respectively.

$B_n(u,m)$ may be expressed in terms of the sine and cosine integrals

$$S(x) = \int_1^\infty \frac{\sin xt}{t}\, dt$$

$$C(x) = \int_1^\infty \frac{\cos xt}{t}\, dt$$

($C(x)$ and $S(x)$ are evaluated by the subroutine CIN.) The resulting formula for B_n is

$$B_n(u,m) = \left\{ \left[\frac{2i}{\ell v}\left(1 + \frac{1}{2} i\ell n - \frac{1}{8}\ell^2 n^2\right) - \frac{1}{24}\ell^2 uv \right] e^{-\frac{1}{2} i\ell v^2/u} \right.$$

$$+ \left[v\left(-\frac{1}{2} i\ell + \frac{1}{4}\ell^2 n\right) + \frac{1}{48} i\ell^3 v^3 \right] \left[C\left(\frac{\ell v^2}{2u}\right) \right. \tag{43}$$

$$\left. \left. - i\, S\left(\frac{\ell v^2}{2u}\right) \right] \right\} \Bigg|_{v = m - \frac{1}{2}}^{m + \frac{1}{2}}$$

To evaluate $C_n(u,m)$, put $v = s + m$. Expanding part of the exponential gives the approximation

$$\int_{m-\frac{1}{2}}^{m+\frac{1}{2}} dv\, e^{-\frac{1}{2} i\ell v^2/u} = \int_{-\frac{1}{2}}^{\frac{1}{2}} ds\, e^{-\frac{1}{2} i\ell (m^2 + 2ms + s^2)/u}$$

$$\cong e^{-\frac{1}{2} i\ell m^2/u} \int_{-\frac{1}{2}}^{\frac{1}{2}} ds\, e^{-i\ell ms/u} \left(1 - \frac{1}{2} i\ell s^2/u\right)$$

and performing the integration gives

$$\int_{m-\frac{1}{2}}^{m+\frac{1}{2}} dv\, e^{-\frac{1}{2} i\ell v^2/u} = e^{-\frac{1}{2} i\ell m^2/u} \left\{ \frac{\sin(\ell m/2u)}{\ell m/2u} \left(1 - \frac{1}{8}\frac{i\ell}{u}\right) \right.$$

$$\left. + \frac{iu}{\ell m^2} \left[\frac{\sin(\ell m/2u)}{\ell m/2u} - \cos(\ell m/2u) \right] \right\}, \quad m \neq 0$$

For small values of $\ell m/2u$, the trigonometric functions of this argument are expanded in power series. To sufficient accuracy,

$$\int_{m-\frac{1}{2}}^{m+\frac{1}{2}} dv\, e^{-\frac{1}{2} i\ell v^2/u} = e^{-\frac{1}{2} i\ell m^2/u} \left[1 - \frac{1}{6}\left(\frac{\ell m}{2u}\right)^2 - \frac{1}{24}\frac{i\ell}{u} \right], \quad \ell m/2u < 0.2$$

It may be verified that this is valid for m = 0.

Combining these expressions,

$$C_n(u,m) = e^{-\frac{1}{2} i\ell m^2/u}\left[-\frac{2}{u}\left(1+\frac{1}{2} i\ell n - \frac{1}{8}\ell^2 n^2\right) + 2\left(-\frac{1}{2} i\ell + \frac{1}{4}\ell^2 n\right) - \frac{1}{12}\ell^2 u\right]$$

$$\cdot \begin{cases} \left(1 - \frac{1}{8}\frac{i\ell}{u}\right)\frac{\sin(\ell m/2u)}{\ell m/2u} + \frac{iu}{\ell m^2}\left[\frac{\sin(\ell m/2u)}{\ell m/2u} - \cos(\ell m/2u)\right], & \ell m/2u > 0.2 \\ 1 - \frac{1}{24}\frac{i\ell}{u} - \frac{1}{6}\left(\frac{\ell m}{2u}\right)^2, & \ell m/2u < 0.2 \end{cases} \tag{44}$$

The subroutine PØT2 evaluates the influence coefficients according to Equations (41), (42), (43), and (44).

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APPENDIX III. COMPUTER PROGRAM LISTINGS

```
      MAIN PROGRAM

      DIMENSION A(2,100,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      DIMENSION G(10)
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      DATA Z/1H /
C
C     INITIALIZATION OF DATA ARRAY
C
      DO 1 I=1,700
    1 DA(I)=0.0
      DO 11 I=1,24
   11 DA(I)=Z
      DO 13 I=104,700,4
   13 DA(I)=1.0
C
   16 CALL DATRD(DA)
      CK=DA(23)*DA(24)/DA(25)*6.28318531
      DI=DA(27)
      L=DI
      D=1.0/DI
      DH=0.5*D
      WRITE
     0  (6, 49)(DA(I),I=1,12)
   36 IF (L) 83,83,37
   37 IF (50-L) 83,38,38
   38 WRITE
     0  (6, 41)L,CK
   41 FORMAT(1H010X,I2,23H BOXES ALONG ROOT CHORD/1H010X,19HREDUCED FREQ
     1UENCY =F6.3)
      WRITE
     0  (6, 42)DA(23)
   42 FORMAT(1H011X,10HFREQUENCY=1PE11.3)
      IF (DA(26)) 33,35,33
   33 DO 34 I=13,22
   34 DA(I)=Z
      GO TO 27
```

```
   35 CALL SHAPE
      CALL FORCI
   27 LIM=ML(L)
      IF (LIM-50) 22,22,101
   22 LIM2=2*LIM
      LPOT=  MINO(L,15)
      CALL POT2(100,LIM2,LPOT,CK,D)
      M=0
      K=DA(28)
      GO TO 4
C
C     PRELIMINARY CALCULATIONS ARE FINISHED.
C     THE NEXT SECTION IS GONE THROUGH FOR EACH MODE.
C
    3 IF (DA(26)) 4,28,4
   28 CALL DATRD(DA)
    4 K=K-1
      M=M+1
      WRITE
     0  (6, 48)M
   48 FORMAT (1H115X,8HMODE NO.I3)
      WRITE
     0  (6, 49)(DA(I),I=13,22)
   49 FORMAT (1H010X,12A6)
      IF (DA(26)) 29,7,29
    7 CALL DRED
    9 G(M)=DA(39)
   29 IF (G(M)) 74,72,74
   72 CALL WVAL
      IF (ML) 24,26,24
   24 LIM2=2*PL
      DO 25 J=1,LIM2
   25 W(J,1,1)=W(J,1,1)*2.0/3.14159265
C     LEADING EDGE CORRECTION
   26 CONTINUE
      CALL BOXP
   74 IF (K) 6,6,5
```

```
    5 IF (M-10) 3,6,6
C
C     FINAL SECTION OF PROGRAM - COMPUTATION OF GENERALIZED FORCES
C
    6 WRITE (6 ,46)
   46 FORMAT (1H110X,18HGENERALIZED FORCES/1H05X,5HMODES/4X,11HOSC.  DEF
     1L.8X,9HREAL PART10X,9HIMAG PART10X,10HABS. VALUE6X,11HPHASE ANGLE)
      AC=8.0/AREA
      DO 12 M1=1,M
      IF (G(M1)) 12,14,12
   14 DO 18 M2=1,M
      S1=0.0
      S2=0.0
      N1=5*(M1-1)
      N2=5*(M2-1)
      DO 8 J=1,5
      J1=J+N1
      J2=J+N2
      DO 8 I=1,5
      S1=S1+PS(1,I,J1)*DF(I,J2)
    8 S2=S2+PS(2,I,J1)*DF(I,J2)
      S1=AC*S1
      S2=AC*S2
      S3= SQRT(S1**2+S2**2)
      S4=  ATANO(S2,S1)
   18 WRITE
     0  (6, 47)M1,M2,S1,S2,S3,S4
   47 FORMAT (1H02I6,1P3E19.5,0P1F16.4)
   12 CONTINUE
      WRITE  ( 6,43)
   43 FORMAT(1H1)
      GO TO 16
C
C     ERROR EXITS
C
   83 IPR=27
   84 WRITE
```

```
      O  (6, 45)IPR                                                     00001530
   45 FORMAT(1H010X,8HBAD DATAI4)                                       00001540
      GO TO 102                                                         00001550
  101 WRITE   ( 6,56)                                                   00001560
   56 FORMAT(1H010X,42HLATERAL LIMIT ON NUMBER OF BOXES EXCEEDED.)      00001580
  102 STOP                                                              00001600
      END                                                               00001610
```

SUBROUTINE SHAPE

```
      SUBROUTINE SHAPE
      DIMENSION A(2,100,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      IEDG=0
      NS=DA(29)
      IF (NS) 81,81,1
    1 IF (NS-3) 2,2,81
    2 NSP=NS+1
      IF (DA(24)) 82,82,3
    3 DO 4 I=1,NSP
      XEDG(I)=DA(2*I+27)/DA(24)
    4 YEDG(I)=DA(2*I+28)/DA(24)
      XEDG=0.0
      XEDG(NS+2)=1.0
      YEDG(NS+2)=YEDG(NS+1)
      Y1=DA(30)
      IF (Y1) 83,20,20
   20 K=0
      X=DH
      AREA=0.0
    6 DO 15 I=1,L
    7 IF (X-XEDG(K+1)) 8,8,9
    8 ML(I)=0.5+DI*(YEDG(K)+F*(X-XEDG(K))/G)
      GO TO 15
    9 K=K+1
      G=XEDG(K+1)-XEDG(K)
      F=YEDG(K+1)-YEDG(K)
      IF (G) 84,12,12
   12 IF (F) 85,13,13
   13 AREA=AREA+G*(YEDG(K+1)+YEDG(K))
      GO TO 7
   15 X=X+D
      IF (XEDG(NS+1)-1.0) 17,16,84
   16 IF (YEDG(NS+1)-YEDG(NS)) 17,17,18
   17 IEDG=1
```

```
      SUBROUTINE SHAPE

   18 RETURN
   81 IPR=29
      GO TO 86
   82 IPR=24
      GO TO 86
   83 IPR=30
      GO TO 86
   84 K=  MINC(K,NS)
      IPR=2*K+29
      GO TO 86
   85 IPR=2*K+30
   86 WRITE
     0  (6, 41)IPR
   41 FORMAT(1H010X,8HBAD DATAI3)
      STOP
      END
```

```
      SUBROUTINE FORCI
C
C     CONTROL SUBROUTINE FOR CALCULATION OF INTEGRALS USED
C     TO FIND GENERALIZED FORCES
C
      SUBROUTINE FORCI
      DIMENSION A(2,100,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      COMMON YMAX2,N
      YMAX2=YEDG(NS+1)**2
      DO 3 J=1,9
      DO 2 I=1,9
    2 AXY(I,J)=0.0
    3 AY(J)=0.0
      IF (DA(30)) 4,5,4
    4 N=0
      CALL SECT
C
C     SECT DOES THE CALCULATIONS FOR EACH SECTION OF THE PLANFORM
C
    5 DO 7 I=1,NS
      IF (YEDG(I)-YEDG(I+1)) 6,7,7
    6 N=I
      CALL SECT
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SECT
C
C     AXY(I,J) IS THE INTEGRAL OVER THE PLANFORM OF
C           X**(I-1)*Y**(2J-2)*P(X,Y)*Q(Y)
C
C     IN TERMS OF THE EQUATION OF THE LEADING EDGE   X = XO(Y)
C           P(X,Y) = SQRTF(X**L-XO**L)
C     WHERE L = 1 OR 2
C           Q(Y) = 1 OR SQRTF(1-(Y/YMAX)**2)
C     DEPENDING ON THE WING SHAPE
C
C     AY(J) IS THE INTEGRAL OVER THE TRAILING EDGE OF
C           Y**(2J-2)*P(1,Y)*Q(Y)
C
      SUBROUTINE SECT
      DIMENSION A(2,100,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      COMMON YMAX2,N
      DIMENSION U(6),H(6)
C
C     COMPUTES THE CONTRIBUTION TO AXY,AY OF THE SECTION OF WING
C     BOUNDED BY A SEGMENT OF THE LEADING EDGE, THE TRAILING EDGE,
C     AND TWO LINES ON WHICH Y IS CONSTANT
C
C     A SIX POINT GAUSSIAN FORMULA IS USED FOR THE INTEGRATION
C     OVER EACH VARIABLE
C
      U(1)=0.61930959
      U(2)=0.83060469
      U(3)=0.96623476
      U(4)=0.03376524
      U(5)=0.16939531
      U(6)=0.38069041
      H(1)=0.23395697
      H(2)=0.18038079
      H(3)=0.08566225
```

```
      SUBROUTINE SECT
      H(4)=H(3)
      H(5)=H(2)
      H(6)=H(1)
C
C     GAUSSIAN POINTS AND WEIGHTS FOR THE INTERVAL (0,1)
C
      X2=XEDG(N+1)
      X1=XEDG(N)
      Y2=YEDG(N+1)
      Y1=YEDG(N)
      IF (N) 4,2,4
    2 X1=0.0
      Y1=0.0
    4 DY=Y2-Y1
      DO 19 J=1,6
      V=U(J)
      G=H(J)*DY
      IF (Y2**2-YMAX2) 7,6,7
    6 G=2.0*V*G
      V=V*V
    7 Y=Y2-V*DY
      XO=X2+V*(X1-X2)
      XOP=1.0-XO
      G=G* SQRT(XOP)
      YQ=Y*Y
      IF (IEDG) 8,9,8
    8 G=G* SQRT(1.0-YQ/YMAX2)
    9 E=2.0*XOP*G
      IF (DA(30)) 10,10,11
   10 G=G* SQRT(1.0+XO)
   11 DO 17 I=1,6
      U2=U(I)**2
      X=XO+XOP*U2
      F=E*H(I)*U2
      IF (DA(30)) 12,12,13
   12 F=F* SQRT(X+XO)
   13 YP=1.0
```

```
      SUBROUTINE SECT

      DO 16 M=1,9
      XP=YP
      DO 15 L=1,9
      AXY(L,M)=AXY(L,M)+XP*F
   15 XP=X*XP
   16 YP=YQ*YP
   17 CONTINUE
      YP=1.0
      DO 18 M=1,9
      AY(M)=AY(M)+YP*G
   18 YP=YQ*YP
   19 CONTINUE
      RETURN
      END
```

SUBROUTINE DRED

```
      SUBROUTINE DRED                                                   00003400
      DIMENSION A(2,1CO,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)      00003410
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)          00003420
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG   00003430
      COMMON AREA,DH                                                    00003440
      DIMENSION IXB(5),B(25),C(25,25),G(25)                             00003450
      COMMON C,B,G,IXB                                                  00003460
      NP=DA(98)                                                         00003470
      IF (NP) 83,24,30                                                  00003480
C                                                                       00003490
C     A POLYNOMIAL FOR THE DEFLECTION IS FITTED TO VALUES               00003500
C     OF DEFLECTION AT GIVEN POINTS.                                    00003510
C                                                                       00003520
   30 NX=DA(99)                                                         00003530
      NY=DA(1CO)                                                        00003540
      IF (NX) 81,81,31                                                  00003550
   31 IF (NY) 82,82,32                                                  00003560
   32 IF (150-NP) 83,38,38                                              00003570
   38 IF (NP-NX) 81,34,34                                               00003580
   34 IF (NP-NY) 82,35,35                                               00003590
   35 MX=  MINO(NX,5)                                                   00003600
      MY=  MINO(NY,5)                                                   00003610
      IY=MY                                                             00003620
      IXY=MX+MY                                                         00003630
      DO 2 J=1,5                                                        00003660
      DO 1 I=1,5                                                        00003662
    1 COE(I,J)=0.0                                                      00003664
    2 IXB(J)=MX                                                         00003670
      NC=MX*MY                                                          00003680
    3 DO 5 I=1,NC                                                       00003690
      DO 4 J=1,NC                                                       00003700
    4 C(I,J)=C.0                                                        00003710
    5 B(I)=0.0                                                          00003720
      KP=100                                                            00003730
      DO 11 IP=1,NP                                                     00003740
      X=DA(KP+1)/DA(24)                                                 00003750
      Y2=(DA(KP+2)/DA(24))**2                                           00003760
```

```
      SUBROUTINE DRED
      DEF=DA(KP+3)
      WT=DA(KP+4)
      IF (WT) 84,84,6
    6 YP=1.0
      K=1
      DO 8 J=1,IY
      XYP=YP
      JX=IXB(J)
      DO 7 I=1,JX
      G(K)=XYP
      XYP=X*XYP
    7 K=K+1
    8 YP=Y2*YP
      DO 10 I=1,NC
      DO 9 J=1,NC
    9 C(I,J)=C(I,J)+G(I)*G(J)*WT
   10 B(I)=B(I)+G(I)*DEF*WT
   11 KP=KP+4
      K=MSIMER(25,NC,1,C,B)
      IF (K-1) 22,22,15
   15 DO 16 I=1,IY
      IP=IY+1-I
      IF (IXB(IP)+IP-IXY) 16,17,17
   16 CONTINUE
      IXY=IXY-1
      GO TO 15
   17 IXB(IP)=IXB(IP)-1
      IF (IXB(IP)) 18,18,19
   18 IY=IP-1
   19 NC=0
      DO 20 I=1,IY
   20 NC=NC+IXB(I)
      GO TO 3
   22 K=1
      DO 23 J=1,IY
      JX=IXB(J)
      DO 23 I=1,JX
```

```
      SUBROUTINE DRED
      COE(I,J)=B(K)
   23 K=K+1
      IF (DA(87)) 61,66,61
   61 WRITE  ( 6,41)
      DO 64 J=1,IY
      JX=IXB(J)
      DO 63 I=1,JX
      WRITE
     0  (6, 42)I,J,COE(I,J)
   63 CONTINUE
   64 CONTINUE
   66 CONTINUE
      GO TO 28
   24 YP=1.0
      K=1
      DO 27 J=1,5
      XYP=YP
      DO 26 I=1,5
      COE(I,J)=XYP*DA(K+45)
      K=K+1
   26 XYP=XYP*DA(24)
   27 YP=YP*DA(24)**2
   28 K=25*(M-1)
      DO 29 I=1,25
      K=K+1
   29 DF(K,1)=COE(I,1)
      RETURN
   81 IPR=99
      GO TO 85
   82 IPR=100
      GO TO 85
   83 IPR=98
      GO TO 85
   84 IPR=K+4
   85 WRITE
     0  (6, 45)IPR
      STOP
```

```
      SUBROUTINE ORED

   41 FORMAT(1H010X,56HCOMPUTED DEFLECTION = SUM OF DEF(N,M)*X**(N-1)*Y*00004610
     1*(2M-2)/1H010X,54H(IN DIMENSIONLESS COORDINATES - DISTANCE/CHORD L00004620
     2ENGTH)/1H09X,1HN7X,1HM16X,8HDEF(N,M))                             00004630
   42 FORMAT(3X,2I8,1PE25.5)                                            00004640
   45 FORMAT(1H010X,8HBAC DATAI4)                                       00004650
      END                                                               00004660
```

SUBROUTINE WVAL

```
      SUBROUTINE WVAL
C
C     EVALUATION OF THE UPWASH ARRAY
C
      DIMENSION A(2,100,16),W(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      COMMON A,W,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      DIMENSION G(5,5),H(5,5)
      COMMON G,H
      J1=5*(M-1)
      DO 3 J=1,5
      J1=J1+1
      CI=1.0
      DO 2 I=1,5
      G(I,J)=CI*DF(I+1,J1)
      H(I,J)=CK*[illegible](I,J1)
    2 CI=CI+1.0
    3 G(5,J)=0.0
C
C     G AND H ARE THE COEFFICIENTS OF THE REAL AND IMAGINARY PARTS
C      OF THE UPWASH
C
      X=DH
      DO 10 I=1,L
      JL=ML(I)
      IF (JL) 5,10,5
    5 Y=DH
      DO 9 J=1,JL
      Y2=Y*Y
      W(1,J,I)=0.0
      W(2,J,I)=0.0
      YP=1.0
      DO 8 J1=1,5
      XYP=YP
      DO 7 I1=1,5
      W(1,J,I)=W(1,J,I)+G(I1,J1)*XYP
```

```
      SUBROUTINE WVAL
      W(2,J,I)=W(2,J,I)+H(I1,J1)*XYP
    7 XYP=X*XYP
    8 YP=Y2*YP
    9 Y=Y+D
   10 X=X+D
      RETURN
      END
```

SUBROUTINE BOXP

```
      SUBROUTINE BOXP
      DIMENSION A(2,100,16),S(2,50,50),DA(700),PS(2,5,50),DF(5,50)
      DIMENSION ML(50),AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5)
      DIMENSION EDG(50),PR(2,50),PSI(2,50),G(20),XO(50),IXB(4)
      DIMENSION C(20,20),B(20,2)
      COMMON A,S,DA,PS,DF,ML,AXY,AY,XEDG,YEDG,COE,M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH
      COMMON B,C,EDG,PR,PSI,G,XO,IXB
C
      IF (DA(88)) 71,75,71
   71 WRITE  ( 6,47)
   47 FORMAT(1H110X,43HTHE UPWASH ARRAY (REAL AND IMAGINARY PARTS))
      DO 74 I=1,L
      JL=ML(I)
      IF (JL) 73,74,73
   73 WRITE
     0  (6, 42)I
      IF (I-1) 77,77,79
   77 DO 78 J=1,JL
      S1=S(1,J,I)*1.57079633
      S2=S(2,J,I)*1.57079633
   78 WRITE
     0  (6, 142)S1,S2
      GO TO 74
   79 WRITE
     0  (6, 142)(S(1,J,I),S(2,J,I),J=1,JL)
  142 FORMAT(1H 1P2E24.5)
   74 CONTINUE
C     THESE ARE THE UPWASHES
C
   75 CONTINUE
      CALL BOXPO
C      BOXPO COMPUTES THE POTENTIAL VALUES IN EACH BOX.
C     THEY ARE STORED IN THE ARRAY S.
C
      IF (DA(89)) 91,95,91
   91 WRITE  ( 6,143)
```

```
      SUBROUTINE BOXP

  143 FORMAT(1H110X,46HTHE POTENTIAL ARRAY (REAL AND IMAGINARY PARTS))
      DO 94 I=1,L
      JL=ML(I)
      IF (JL) 93,94,93
   93 WRITE
     0  (6, 42)I
      WRITE
     0  (6, 142)(S(1,J,I),S(2,J,I),J=1,JL)
   94 CONTINUE
C     THESE ARE THE POTENTIALS
C
   95 CONTINUE
C     FIT OF A SERIES TO THE POTENTIAL VALUES
C
      JL=ML(L)
      DY=D/YEDG(NS+1)
      Y=0.5*DY
      DO 201 J=1,JL
      IF (IEDG) 202,203,202
  202 EDG(J)= SQRT(1.0-Y*Y)
      Y=Y+DY
      GO TO 201
  203 EDG(J)=1.0
  201 CONTINUE
      N=0.5+DI*YEDG
      IF (N) 4,4,2
    2 DO 3 I=1,N
    3 XO(I)=0.0
    4 X1=0.0
      N1=N
      Y1=YEDG
      DO 8 K=1,NS
      X2=XEDG(K+1)
      Y2=YEDG(K+1)
      N=DI*Y2+0.5
      IF (N1-N) 5,7,7
    5 N1=N1+1
```

```
      SUBROUTINE BOXP

      DO 6 I=N1,N
      Y=D*( FLOAT(I)-0.5)
    6 XO(I)=X1+(X2-X1)*(Y-Y1)/(Y2-Y1)
    7 X1=X2
      Y1=Y2
    8 N1=N
      AS=0.0
      DO 119 I=1,L
      JL=ML(I)
      IF (JL) 119,119,217
  217 DO 118 J=1,JL
      AS=AMAX1(AS, ABS(S(1,J,I)), ABS(S(2,J,I)))
  118 CONTINUE
  119 CONTINUE
      IX=5
      IY=4
      IXY=9
   13 DO 14 I=1,IY
   14 IXB(I)=IX
   16 NC=0
      DO 17 I=1,IY
   17 NC=NC+IXB(I)
      DO 19 I=1,NC
      DO 18 J=1,NC
   18 C(I,J)=0.0
      B(I,1)=0.0
   19 B(I,2)=0.0
      X1=0.5*D
      DO 25 I=1,L
      JL=ML(I)
      Y=0.5*D
      IF (JL) 25,25,20
   20 DO 24 J=1,JL
      XR=X1-XO(J)
      IF (YEDG) 601,601,602
  601 XR=XR*(X1+XO(J))
  602 XR= SQRT(XR)
```

```
      SUBROUTINE BOXP

      YP=1.0
      K=1
      DO 23 N1=1,IY
      XP=XR*YP
      JX=IXB(N1)
      DO 22 N=1,JX
      G(K)=XP*EDG(J)
      XP=X1*XP
   22 K=K+1
   23 YP=Y*Y*YP
      Y=Y+D
      DO 24 N1=1,NC
      DO 124 N=1,NC
  124 C(N1,N)=C(N1,N)+G(N1)*G(N)
      DO 24 N=1,2
   24 B(N1,N)=B(N1,N)+G(N1)*S(N,J,I)
   25 X1=X1+D
      K=MSIMER(20,NC,2,C,B)
      IF (K-1) 30,30,29
C
C     IF XSIMEQ FAILS, THE FOLLOWING SECTION REDUCES THE NUMBER OF TERMS
C      IN THE SERIES.
C
   29 DO 61 I=1,IY
      IP=IY+1-I
      IF (IXB(IP)+IP-IXY) 61,62.62
   61 CONTINUE
      IXY=IXY-1
      GO TO 29
   62 IXB(IP)=IXB(IP)-1
      IF (IXB(IP)) 63,63,16
   63 IY=IP-1
      GO TO 16
C
   30 K=1
      AC=0.0
      DO 133 I=1,IY
```

```
      SUBROUTINE BOXP

      JX=IXB(I)
      CJ=1.0
      DO 33 J=1,JX
      C(K,1)=B(K,1)
      C(K,2)=B(K,2)
      AC=AMAX1(AC, ABS(C(K,1)), ABS(C(K,2)))
      IF (J.EQ.1) GO TO 33
      B(K-1,1)=CJ*C(K,1)
      B(K-1,2)=CJ*C(K,2)
      CJ=CJ+1.0
   33 K=K+1
      B(K-1,1)=0.0
  133 B(K-1,2)=0.0
      IF (AC-50.0*AS) 117,117,29
C
C     GO TO 29 IF COEFFICIENTS ARE TOO LARGE.
C
  117 IF (DA(90)) 110,113,110
C
C     PRINTOUT OF COEFFICIENTS OF POTENTIAL
C
  110 WRITE  ( 6,44)
      IF (YEDG) 603,603,605
  603 WRITE  ( 6,45)
      IF (IEDG) 604,607,604
  604 WRITE  ( 6,48)
      GO TO 607
  605 WRITE  ( 6,145)
      IF (IEDG) 606,607,606
  606 WRITE  ( 6,148)
  607 WRITE  ( 6,46)
      K=1
      DO 111 I=1,IY
      JL=IXB(I)
      DO 111 J=1,JL
      WRITE (6,49) J,I,C(K,1),C(K,2)
  111 K=K+1
```

```
      SUBROUTINE BOXP
      WRITE  ( 6,149)
  113 IF (DA(91)) 114,116,114
C
C     PRINTOUT OF VALUES OF POTENTIAL AND PRESSURE
C
  114 WRITE  ( 6,41)
   41 FORMAT(1H010X,9HPOTENTIAL45X,8HPRESSURE)
      X1=0.5*D
      DO 39 I=1,L
      JL=ML(I)
      Y=0.5*D
      IF (JL) 39,39,34
   34 DO 38 J=1,JL
      XR=X1-XO(J)
      XQ=0.5
      IF (YEDG)  608,608,609
  608 XQ=X1
      XR=XR*(X1+XO(J))
  609 XQ=XQ/XR
      XR= SQRT(XR)
      DO 37 N=1,2
      PSI(N,J)=0.0
      PR(N,J)=0.0
      K=1
      YP=EDG(J)
      DO 37 N1=1,IY
      XP1=XR*YP
      JX=IXB(N1)
      DO 36 M1=1,JX
      PSI(N,J)=PSI(N,J)+C(K,N)*XP1
      PR(N,J)=PR(N,J)+B(K,N)*XP1
      XP1=X1*XP1
   36 K=K+1
   37 YP=Y*Y*YP
      PR(1,J)=2.0*(PR(1,J)+PSI(1,J)*XQ-PSI(2,J)*CK)
      PR(2,J)=2.0*(PR(2,J)+PSI(2,J)*XQ+PSI(1,J)*CK)
   38 Y=Y+D
```

```
      SUBROUTINE BOXP
      WRITE
     0  (6, 42)I
   42 FORMAT(1H010X,I2,6HTH ROW)
      WRITE
     0  (6, 43)(PSI(1,J),PSI(2,J),PR(1,J),PR(2,J),J=1,JL)
   43 FORMAT(1H 1P2E24.5,5X,2E24.5)
   39 X1=X1+D
C
  116 JM0=5*(M -1)
      CJ=0.0
      DO 172 I=1,5
      DO 72 J=1,5
      J0=J+JM0
      PS (1,I,J0)=0.0
      PS (2,I,J0)=0.0
      K=1
      DO 72 K2=1,IY
      J2=J+K2-1
      JX=IXB(K2)
      DO 72 K1=1,JX
      J1=K1+I-1
      X1=AY(J2)-CJ*AXY(J1-1,J2)
      Y=AXY(J1,J2)*CK
      PS (1,I,J0)=PS (1,I,J0)+C(K,1)*X1-C(K,2)*Y
      PS (2,I,J0)=PS (2,I,J0)+C(K,1)*Y +C(K,2)*X1
   72 K=K+1
  172 CJ=CJ+1.0
      RETURN
C
   44 FORMAT(1H-10X,53HPOTENTIAL = SUM OF PO(M,N)*X**(M-1)*Y**(2N-2)*SQR
     1TF(X)
   45 FORMAT(1H+63X,10H**2-XO**2))
  145 FORMAT(1H+63X,4H-XO))
   48 FORMAT(1H+73X,21H*SQRTF(1-(Y/YMAX)**2))
  148 FORMAT(1H+67X,21H*SQRTF(1-(Y/YMAX)**2))
   46 FORMAT(1H015X,52HWHERE X = XO(Y) IS THE EQUATION OF THE LEADING ED
     1GE./1H020X,21HCOEFFICIENTS  PO(M,N)/1H07X,1HM7X,1HN14X,9HREAL PART
```

SUBROUTINE BOXP

```
      216X,10HIMAG. PART)
   49 FORMAT(1H 2I8,1P2E25.5)
  149 FORMAT(1H1)
      END
```

```
      SUBROUTINE BOXPO
C     SOLUTION OF SIMULTANEOUS EQUATIONS FOR THE POTENTIAL
      COMMON A(2,100,16),S(2,50,50),DA(700),PS(2,5,50),DF(5,50),ML(50)
      COMMON AXY(9,9),AY(9),XEDG(5),YEDG(5),COE(5,5),M,L,NS,D,DI,CK,IEDG
      COMMON AREA,DH,E(2,50,50)
      I1=0
      DO 9 I=1,L
      KO=MAXO(1,I-14)
      JL=ML(I)
      IF (JL.EQ.0) GO TO 9
      IF (I1.EQ.0) GO TO 6
C     SUBTRACTION OF CONTRIBUTIONS OF PRECEDING ROWS TO UPWASH
      DO 5 J=1,JL
      DO 5 K=KO,I1
      KL=ML(K)
      K1=I+1-K
      IF (KL.EQ.0) GO TO 5
      DO 4 N=1,KL
      N1=N+J
      N2=IABS(N-J)+1
      A1=A(1,N1,K1)+A(1,N2,K1)
      A2=A(2,N1,K1)+A(2,N2,K1)
      S(1,J,I)=S(1,J,I)-A1*S(1,N,K)+A2*S(2,N,K)
    4 S(2,J,I)=S(2,J,I)-A2*S(1,N,K)-A1*S(2,N,K)
    5 CONTINUE
C     SETTING UP MATRIX FOR SIMULTANEOUS EQUATIONS
    6 DO 8 J=1,JL
      DO 8 K=1,J
      N1=J+K
      N2=IABS(J-K)+1
      E(1,J,K)=A(1,N1,1)+A(1,N2,1)
      E(2,J,K)=A(2,N1,1)+A(2,N2,1)
      E(1,K,J)=E(1,J,K)
    8 E(2,K,J)=E(2,J,K)
C     SOLUTION OF EQUATIONS
      K=MSIMEC(50,JL,1,E,S(1,1,I))
      IF (K.NE.1) GO TO 12
    9 I1=I1+1
```

```
      SUBROUTINE BOXPO

      RETURN
   12 WRITE ( 6,41)
   41 FORMAT(1H010X,59HSOLUTION OF SIMULTANEOUS EQUATIONS FOR THE POTENT
     1IAL FAILED)
      STOP
      END
```

```
      SUBROUTINE POT2

C
C     THE VELCCITY FIELD OF A UNIFORM DOUBLET DISTRIBUTION
C     OVER A BOX IS COMPUTED AT ALL POINTS AT WHICH IT WILL BE
C     NEEDED AND STORED IN THE ARRAY A IN COMMON
C
C     MO,NO CONTROL THE NUMBER OF VALUES COMPUTED
C
C     M2 IS THE RANGE OF THE SECOND SUBSCRIPT IN THE ARRAY,
C     DIMENSIONED A(2,M2,N2), BUT TREATED HERE AS AN ARRAY
C     WITH TWO SUBSCRIPTS
C
      SUBROUTINE POT2(M2,MO,NO,CK,D)
      DIMENSION A(2,1)
      COMMON A
      M=MO
      N=NO
      DK=CK*D
      DK2=DK**2
      M1=M-1
      DK8=DK2/8.0
      DK4=2.0*DK8
      DK12=DK2/12.0
      CM=0.5
      DH=DK*0.5
      DM=0.5*DH
      DD=2.0*CK
      DDM=DD
      D1=0.25*DK2
      B5=DK2/24.0
      DO 3 I=1,M
      B1=0.0
      B4=2.0/DM
      B2=B5/B4-DH
      B3=-0.5*B5
      D3=DH*B4+B5
      D4=DK8*B4
      DD4=2.0*D4
```

```
      SUBROUTINE POT2

      CN=1.0
      K=I
      C3=0.0
      C4=0.0
      C7=0.0
      C8=0.0
      DO 2 J=1,N
      A1=DM/CN
      C1=CM* COS(A1)
      C2=-CM* SIN(A1)
      C5=CM*CIN(A1,C6)
      C6=-CM*C6
      C9=C1-C3
      C10=C2-C4
      C11=C5-C7
      C12=C6-C8
      A(1,K)=B3*C9-B4*C10-B5*C3-B1*C11-B2*C12
      A(2,K)=B4*C9+B3*C10-B5*C4+B2*C11-B1*C12
   23 C3=C1
      C4=C2
      C7=C5
      C8=C6
      B1=B1-D1
      B3=B3-D3
      B4=B4-D4
      D4=D4+DD4
      CN=CN+2.0
    2 K=K+M2
      CM=CM+1.0
      DM=DM+DDM
    3 DDM=DDM+DD
      DO 5 L=1,2
      K1=1
      DO 5 J=1,N
      DO 4 I=1,M1
      K=K1+M-I
    4 A(L,K)=A(L,K)-A(L,K-1)
```

```
      SUBROUTINE POT2
      A(L,K1)=2.0*A(L,K1)
    5 K1=K1+M2
      CM=0.0
      DM=0.0
      DDM=DK
      DO 12 I=1,M
      C7=0.0
      C8=0.0
      C9=0.0
      C10=0.0
      P1=0.0
      P2=0.0
      CN=1.0
      B6=0.5*DK12
      K=I
      DO 10 J=1,N
      A1=CM/CN
      A2=DM/CN
      IF (A1-0.2) 7,7,8
    7 B1=2.0-A1**2/3.0
      B2=-DK/(6.0*CN)
      GO TO 9
    8 B3= SIN(A1)/A1
      B1=2.0*B3
      B2=(B3- COS(A1))/A2-DH/CN*B3
    9 B3= COS(A2)/CN
      B4= SIN(A2)/CN
      C3=B1*B3+B2*B4
      C4=B2*B3-B1*B4
      B5=DH*CN
      C1=B5*C4-2.0*C3
      C2=-2.0*C4-B5*C3
      C5=C1-C7
      C6=C2-C8
      P3=P2-B6*CN
      P4=P3+2.0*DK12*(CN-1.0)
      A(1,K)=A(1,K)+C5-P1*C6+P3*C3-P4*C9
```

```
      SUBROUTINE POT2
      A(2,K)=A(2,K)+C6+P1*C5+P3*C4-P4*C10                               00009080
      P1=P1+DH                                                          00009090
      P2=P2+CN*DK4                                                      00009100
      CN=CN+2.0                                                         00009110
      C7=C1                                                             00009120
      C8=C2                                                             00009130
      C9=C3                                                             00009140
      C10=C4                                                            00009150
      B6=B6+DK12                                                        00009160
   10 K=K+M2                                                            00009170
      CM=CM+DK                                                          00009180
      DM=DM+DDM                                                         00009190
   12 DDM=DDM+DD                                                        00009200
      D3=CK/(2.0*3.14159265)                                            00009210
      M1=M2-M                                                           00009220
      K=1                                                               00009230
      A1=0.0                                                            00009240
      DO 14 J=1,N                                                       00009250
      C1=D3* SIN(A1)                                                    00009260
      C2=-D3* COS(A1)                                                   00009270
      DO 13 I=1,M                                                       00009280
      DF     =A(1,K)*C1+A(2,K)*C2                                       00009290
      A(2,K)=A(2,K)*C1-A(1,K)*C2                                        00009300
      A(1,K)=DF                                                         00009310
   13 K=K+1                                                             00009320
      K=K+M1                                                            00009330
   14 A1=A1+DH                                                          00009340
      RETURN                                                            00009350
      END                                                               00009360
```

CIN(X,S)

```
C     SINE AND COSINE INTEGRAL SUBROUTINE
C
C     IF CALLED BY THE STATEMENT  C=CIN(X,S)
C     C AND S ARE THE INTEGRALS OVER T FROM 1 TO INFINITY OF
C         COS(XT)/T  AND  SIN(XT)/T
C
      FUNCTION CIN(X1,S)
      SG=1.0
      X=X1
      IF (X) 1,2,2
    1 SG=-SG
      X=-X
    2 X2=X*X
      IF (X-1.0) 3,3,4
C
C     FOR ABS(X) LESS THAN 1 A SERIES EXPANSION IS USED
C
    3 V=(((X2/98.0-0.6)*.05*X2+1.0)*X2/18.0-1.0)*X+1.57079633
      U=((X2/45.0-1.0)*X2/24.0+1.0)*X2/4.0-.577215665-ALOG(X)
      GO TO 5
C
C     FOR ABS(X) GREATER THAN 1 APPROXIMATIONS OF HASTINGS ARE USED
C
    4 P=(((X2+19.394119)*X2+47.411538)*X2+8.493336)/((((X2+21.361055)
     1  *X2+70.376496)*X2+30.038227)*X)
      Q=(((X2+21.383724)*X2+49.719775)*X2+5.089504)/((((X2+27.177958)
     1  *X2+119.918932)*X2+76.707876)*X2)
      CO=COS (X)
      SI=SIN (X)
      U=Q*CO-P*SI
      V=P*CO+C*SI
    5 S=V*SG
      CIN=U
      RETURN
      END
```

DATRD

```
C                CARD-READ SUBROUTINE 'DATRD(DATA(I))'                   00021410
      SUBROUTINE DATRD(DATA)                                             00021420
      DIMENSION DRBU(14),DATA(1)                                         00021430
      DATA ATEST/5HALPHA/,DTEST/1H /,ETEST/1H-/                          00021435
    1 READ   ( 9,2) EMIN,ALP,IND,(DRBU(I),I=1,12)                        00021450
    2 FORMAT(A1,A5,I6,12A6)                                              00021455
      IF (ALP.EQ.ATEST) GO TO 9                                          00021460
C                                                                        00021465
      WRITE (99,2) EMIN,ALP,IND,(DRBU(I),I=1,12)                         00021470
C      CARD IS WRITTEN IN INTERNAL BUFFER                                00021475
      REWIND 99                                                          00021480
      IF (ALP.NE.DTEST) GO TO 8                                          00021490
C                                                                        00021500
C     NUMERIC CARD                                                       00021510
C                                                                        00021520
      READ (99,990) (DRBU(I),I=1,5)                                      00021530
      REWIND 99                                                          00021540
      DO 5 I=1,5                                                         00021550
      IF(DRBU(I))4,6,4                                                   00021560
 4    DATA(IND)=DRBU(I)                                                  00021570
 5    IND=IND+1                                                          00021580
      GOTO 11                                                            00021590
C                                                                        00021600
C     TEST FOR BLANK FIELD                                               00021610
 6    IF(SIGN (1.0,DRBU(I)))5,5,4                                        00021620
C                                                                        00021630
C     ALPHA CARD                                                         00021640
C                                                                        00021650
 9    DO 10 I=1,10                                                       00021770
      DATA(IND)=DRBU(I)                                                  00021780
 10   IND=IND + 1                                                        00021790
   11 IF (EMIN.NE.ETEST) GO TO 1                                         00021800
C                                                                        00021805
C     RETURN IF COLUMN 1 CONTAINS A MINUS SIGN                           00021810
C                                                                        00021815
 13   RETURN                                                             00021820
C                                                                        00021825
```

```
      DATRD
C     BAD CARD
C
    8 READ (99,992) DRBU
      WRITE  ( 6,993) DRBU
      WRITE  ( 6,991)
      REWIND 99
      STOP
  990 FORMAT(12X,5E12.0)
  991 FORMAT(38H BAD DATA ON THIS CARD. JOB TERMINATED  )
  992 FORMAT(14A6)
  993 FORMAT(12H0CARD IMAGE=14A6)
      END
```

SIMULTANEOUS EQUATION SUBROUTINE

```
$IBMAP SIMR                    K=MSIMER(N,L,LB,A,B)                     00050000
*       SOLVES THE SYSTEM OF EQUATIONS  A*X=B.                          00050020
*       TO USE, SET K=MSIMER(N,L,LB,A,B)                                00050030
*         WHERE N IS THE NUMBER OF ROWS FOR WHICH A IS                  00050040
*         DIMENSIONED, AND L IS THE NUMBER OF EQUATIONS.                00050050
*         LB IS THE NUMBER OF COLUMNS IN B.                             00050060
*       K=1 DENOTES SUCCESSFUL SOLUTION                                 00050070
*       K=2 FOR A SINGULAR OR ILL-CONDITIONED MATRIX                    00050080
*       K=3 IF IMPROPER DATA IS GIVEN.                                  00050090
*         TO AVOID THIS SIGNAL, L MUST BE POSITIVE AND AT MOST 100,     00050100
*         N MUST NOT BE LESS THAN L, A MUST NOT INCLUDE A ROW           00050110
*         OF ZEROS.                                                     00050120
*       A IS DESTROYED.                                                 00050130
*       IF K=1, THE SOLUTION IS RETURNED IN B                           00050140
*                                                                       00050150
*      ENTRY                                                            00050160
*                                                                       00050170
MSIMER SAVE    1,2,3,4,5,6,7                                            00050180
*                                                                       00050190
*      PROLOGUE                                                         00050200
*                                                                       00050210
       CLA*    3,4                                                      00050220
       PAX     0,1                                                      00050230
       TXL     E1,1,0                                                   00050240
       TXH     E1,1,100                                                 00050250
       PCD     0,1                                                      00050260
       STD     A4-1                                                     00050270
       STD     A6-1                                                     00050280
       STD     2A6-1                                                    00050290
       STD     A16                                                      00050300
       STD     A16+1                                                    00050310
       STD     2A21                                                     00050320
       STD     A12+1                                                    00050330
       STD     2A26                                                     00050340
       STD     2A26+1                                                   00050350
       STD     2A37                                                     00050360
       STD     2A37+1                                                   00050370
```

SIMULTANEOUS EQUATION SUBROUTINE

```
        STD     A12+2                                   00050380
        STD     A20                                     00050390
        STD     A22                                     00050400
        STD     A25+1                                   00050410
        STD     A25+2                                   00050420
        STD     A33                                     00050430
        STD     A36+1                                   00050440
        STD     A36+2                                   00050450
        SXD     A52,1                                   00050460
        TXI     *+1,1,1                                 00050470
        SCD     A32,1                                   00050480
        TXI     *+1,1,-2                                00050490
        SCD     A22+1,1                                 00050500
        CLA*    5,4                                     00050510
        PAX     0,7                                     00050520
        SXA     1A6-1,7                                 00050530
        SXA     A14,7                                   00050540
        SXA     1A21-1,7                                00050550
        SXA     A26,7                                   00050560
        SXA     A37,7                                   00050570
        CLA*    4,4                                     00050580
        PAX     0,1                                     00050590
        TXL     E1,1,0                                  00050600
A52     TXH     E1,1,**                                 00050610
        SXA     A2,1                                    00050620
        SXA     A5-1,1                                  00050630
        SXD     A9,1                                    00050640
        SXD     A12,1                                   00050650
        TXI     *+1,1,-1                                00050660
        SXD     A7,1                                    00050670
        SXD     A36,1                                   00050680
        SXD     A18,1                                   00050690
        SXD     A38-1,1                                 00050700
        SXD     A21-1,1                                 00050710
        SXD     A23,1                                   00050720
        SXD     A25,1                                   00050730
        SXD     A31,1                                   00050740
```

SIMULTANEOUS EQUATION SUBROUTINE

```
        SCD     A51,1                                   00050750
        SCD     A51+1,1                                 00050760
        TXI     *+1,1,1                                 00050770
        CLA     6,4                                     00050780
        PAC     0,3                                     00050790
        CLA     7,4                                     00050800
        PAC     0,5                                     00050810
A51     TXI     *+1,3, -L+1                             00050820
        TXI     *+1,5, -L+1                             00050830
*                                                       00050840
*       NORMALIZATION OF ROWS                           00050850
*                                                       00050860
A2      AXT     L,2                                     00050870
        PXA     0,3                                     00050880
        PAX     0,6                                     00050890
        PAX     0,7                                     00050900
        PXA     0,0                                     00050910
A3      LDQ     A,6                                     00050920
        LRS     0                                       00050930
        TLQ     *+2                                     00050940
        XCA                                             00050950
        TNX     A4,2,1                                  00050960
        TXI     A3,6, -N                                00050970
A4      TZE     E1                                      00050980
        STO     T                                       00050990
        CLA     =1.0                                    00051000
        FDP     T                                       00051010
        STQ     T                                       00051020
        AXT     L,2                                     00051030
A5      FMP     A,7                                     00051040
        STO     A,7                                     00051050
        LDQ     T                                       00051060
        TNX     A6,2,1                                  00051070
        TXI     A5,7, -N                                00051080
A6      PXA     0,5                                     00051090
        PAX     0,6                                     00051100
        AXT     LB,4                                    00051110
```

SIMULTANEOUS EQUATION SUBROUTINE

```
1A6    FMP    B,6                                   00051120
       STO    B,6                                   00051130
       LDQ    T                                     00051140
       TNX    2A6,4,1                               00051150
       TXI    1A6,6, -N                             00051160
2A6    TNX    A7,1,1                                00051170
       TXI    *+1,3,1                               00051180
       TXI    A2,5,1                                00051190
A7     TXH    B7,2,L -1                             00051200
*                                                   00051210
*      SEARCH FOR MAXIMUM PIVOT IN COLUMN           00051220
*                                                   00051230
       PXA    0,2                                   00051240
       PAX    0,1                                   00051250
       PXA    0,3                                   00051260
       PAX    0,6                                   00051270
       PXA    0,0                                   00051280
A8     LDQ    A,6                                   00051290
       LRS    0                                     00051300
       TLQ    *+3                                   00051310
       XCA                                          00051320
       SXA    A10,1                                 00051330
       TXI    *+1,1,1                               00051340
A9     TXH    *+2,1,L                               00051350
       TXI    A8,6,-1                               00051360
       LDQ    TOL                                   00051370
       TLQ    *+2                                   00051380
       TRA    E3                                    00051390
A10    AXT    **,1                                  00051400
       SXD    *+1,2                                 00051410
       TNX    A17,1,**                              00051420
*                                                   00051430
*      ROW INTERCHANGE                              00051440
*                                                   00051450
       PXA    0,3                                   00051460
       PAX    0,6                                   00051470
       PAX    0,7                                   00051480
```

SIMULTANEOUS EQUATION SUBROUTINE

```
        SCD     *+1,1                                               00051490
        TXI     *+1,7,**                                            00051500
        PXA     0,2                                                 00051510
        PAX     0,4                                                 00051520
A11     CLA     A,6                                                 00051530
        LDQ     A,7                                                 00051540
        STO     A,7                                                 00051550
        STQ     A,6                                                 00051560
        TXI     *+1,4,1                                             00051570
A12     TXH     A13,4,L                                             00051580
        TXI     *+1,6, -N                                           00051590
        TXI     A11,7, -N                                           00051600
A13     PXA     0,5                                                 00051610
        PAX     0,6                                                 00051620
        PAX     0,7                                                 00051630
A14     AXT     LB,4                                                00051640
        SCD     *+1,1                                               00051650
        TXI     *+1,6,**                                            00051660
A15     CLA     B,7                                                 00051670
        LDQ     B,6                                                 00051680
        STO     B,6                                                 00051690
        STQ     B,7                                                 00051700
        TNX     A17,4,1                                             00051710
A16     TXI     *+1,6, -N                                           00051720
        TXI     A15,7, -N                                           00051730
*                                                                   00051740
*       DIVISION OF ROW BY PIVOT                                    00051750
*                                                                   00051760
A17     CLA     =1.0                                                00051770
        FDP     A,3                                                 00051780
        STQ     AM                                                  00051790
A18     TXM     A21,2,L -1                                          00051800
        PXA     0,2                                                 00051810
        PAX     0,4                                                 00051820
        PXA     0,3                                                 00051830
        PAX     0,6                                                 00051840
A20     TXI     *+1,6, -N                                           00051850
```

SIMULTANEOUS EQUATION SUBROUTINE

```
        LDQ     A,6                                 00051860
        FMP     AM                                  00051870
        STO     A,6                                 00051880
        TXI     *+1,4,1                             00051890
        TXL     A20,4,L -1                          00051900
A21     PXA     0,5                                 00051910
        PAX     0,6                                 00051920
        AXT     LB,4                                00051930
1A21    LDQ     B,6                                 00051940
        FMP     AM                                  00051950
        STO     B,6                                 00051960
        TNX     *+2,4,1                             00051970
2A21    TXI     1A21,6, -N                          00051980
*                                                   00051990
*      ROW REDUCTION                                00052000
*                                                   00052010
        PXA     0,2                                 00052020
        PAX     0,1                                 00052030
        PAX     0,4                                 00052040
        TNX     A31,1,1                             00052050
        PXA     0,3                                 00052060
        PAX     0,6                                 00052070
        PAX     0,7                                 00052080
        STA     A29                                 00052090
        SXA     A26+1,5                             00052100
        SXA     3A26,5                              00052110
        TXI     *+1,6,1                             00052120
A22     TXI     *+1,7, -N                           00052130
        TXI     *+1,3, -N+1                         00052140
        SXA     A28,7                               00052150
A23     TXH     A26,2,L -1                          00052160
        SXA     A27,3                               00052170
A24     LDQ     A,6                                 00052180
        FMP     A,7                                 00052190
        CHS                                         00052200
        FAD     A,3                                 00052210
        STO     A,3                                 00052220
```

SIMULTANEOUS ECUATION SUBROUTINE

```
        TXI     *+1,4,1                                          00052230
A25     TXH     A27,4,L -1                                       00052240
        TXI     *+1,3, -N                                        00052250
        TXI     A24,7, -N                                        00052260
A27     AXT     **,3                                             00052270
A26     AXT     LB,4                                             00052280
        AXT     **,7                                             00052290
        TXI     *+1,7,1                                          00052300
        SXA     *-2,7                                            00052310
1A26    LDQ     A,6                                              00052320
        FMP     B,5                                              00052330
        CHS                                                      00052340
        FAD     B,7                                              00052350
        STO     B,7                                              00052360
        TNX     3A26,4,1                                         00052370
2A26    TXI     *+1,5, -N                                        00052380
        TXI     1A26,7, -N                                       00052390
3A26    AXT     **,5                                             00052400
        TNX     A29,1,1                                          00052410
A28     AXT     **,7                                             00052420
        PXA     0,2                                              00052430
        PAX     0,4                                              00052440
        TXI     *+1,3,1                                          00052450
        TXI     A23,6,1                                          00052460
A29     AXT     **,3                                             00052470
A31     TXH     A43,2,L -1                                       00052480
        PXA     0,2                                              00052490
        PAX     0,1                                              00052500
        PAX     0,4                                              00052510
        PXA     0,3                                              00052520
        PAX     0,6                                              00052530
        PAX     0,7                                              00052540
A32     TXI     *+1,3, -N-1                                      00052550
        TXI     *+1,6,-1                                         00052560
A33     TXI     *+1,7, -N                                        00052570
        SXA     A37+1,5                                          00052580
        SXA     A41,5                                            00052590
```

SIMULTANEOUS EQUATION SUBROUTINE

```
        SXA     A40,3                                   00052600
        SXA     A39,7                                   00052610
A34     SXA     A38,3                                   00052620
A35     LDQ     A,6                                     00052630
        FMP     A,7                                     00052640
        CHS                                             00052650
        FAD     A,3                                     00052660
        STO     A,3                                     00052670
        TXI     *+1,4,1                                 00052680
A36     TXH     A37,4,L -1                              00052690
        TXI     *+1,3, -N                               00052700
        TXI     A35,7, -N                               00052710
A37     AXT     LB,4                                    00052720
        AXT     **,7                                    00052730
        TXI     *+1,7,-1                                00052740
        SXA     *-2,7                                   00052750
1A37    LDQ     A,6                                     00052760
        FMP     B,5                                     00052770
        CHS                                             00052780
        FAD     B,7                                     00052790
        STO     B,7                                     00052800
        TNX     A41,4,1                                 00052810
2A37    TXI     *+1,5, -N                               00052820
        TXI     1A37,7, -N                              00052830
A41     AXT     **,5                                    00052840
        TXI     *+1,1,1                                 00052850
        TXH     A40,1,L -1                              00052860
A38     AXT     **,3                                    00052870
A39     AXT     **,7                                    00052880
        PXA     0,2                                     00052890
        PAX     0,4                                     00052900
        TXI     *+1,3,-1                                00052910
        TXI     A34,6,-1                                00052920
A40     AXT     **,3                                    00052930
        TXI     *+1,5,-1                                00052940
        TXI     A7,2,1                                  00052950
A43     CLA     =1                                      00052960
```

SIMULTANEOUS EQUATION SUBROUTINE

```
         TRA      MSIMER+1
*
*       BRANCH FOR LAST ROW
*
B7       CLA      A,3
         SSP
         LDQ      TOL
         TLQ      A17
*
*       ERROR BRANCHES
*
E3       CLA      =2
         TRA      MSIMER+1
E1       CLA      =3
         TRA      MSIMER+1
*
*       STORAGE
*
TOL      OCT      1514000000000
AP       EQU      E.1
T        EQU      E.2
A        EQU      0
B        EQU      0
L        EQU      0
N        EQU      0
LB       EQU      0
         END
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
$IBMAP SIMC                 K=MSIMEC(N,L,LB,A,B)
*      SOLVES THE SYSTEM OF COMPLEX EQUATIONS A*X=B.
*      TO USE, SET K=MSIMEC(N,L,LB,A,B)
*        WHERE N IS THE NUMBER OF ROWS FOR WHICH A IS
*        DIMENSIONED, AND L IS THE NUMBER OF EQUATIONS.
*        LB IS THE NUMBER OF COLUMNS IN B.
*      K=1 DENOTES SUCCESSFUL SOLUTION
*      K=2 FOR A SINGULAR OR ILL-CONDITIONED MATRIX
*      K=3 IF IMPROPER DATA IS GIVEN.
*        TO AVOID THIS SIGNAL, L MUST BE POSITIVE AND AT MOST 100,
*        N MUST NOT BE LESS THAN L, A MUST NOT INCLUDE A ROW
*        OF ZEROS.
*      A IS DESTROYED.
*      IF K=1, THE SOLUTION IS RETURNED IN B
*
*     ENTRY
*
MSIMEC SAVE    1,2,3,4,5,6,7
*
*     PROLOGUE
*
       CLA*    3,4
       ALS     1
       PAX     0,1
       TXL     E1,1,1
       TXH     E1,1,200
       PCD     0,1
       STD     A4-1
       STD     A5-1
       STD     2A6-1
       STD     A16
       STD     A16+1
       STD     2A21
       STD     A12+1
       STD     2A26
       STD     2A26+1
       STD     2A37
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
        STD     2A37+1                                      00053620
        STD     A12+2                                       00053630
        STD     A20                                         00053640
        STD     A22                                         00053650
        STD     A25+1                                       00053660
        STD     A25+2                                       00053670
        STD     A33                                         00053680
        STD     A36+1                                       00053690
        STD     A36+2                                       00053700
        SXD     A52,1                                       00053710
        TXI     *+1,1,2                                     00053720
        SCD     A32,1                                       00053730
        TXI     *+1,1,-4                                    00053740
        SCD     A22+1,1                                     00053750
        CLA*    5,4                                         00053760
        PAX     0,7                                         00053770
        SXA     1A6-1,7                                     00053780
        SXA     A14,7                                       00053790
        SXA     1A21-1,7                                    00053800
        SXA     A26,7                                       00053810
        SXA     A37,7                                       00053820
        CLA*    4,4                                         00053830
        ALS     1                                           00053840
        PAX     0,1                                         00053850
        TXL     E1,1,1                                      00053860
A52     TXH     E1,1,**                                     00053870
        SXA     A2,1                                        00053880
        SXA     A5-1,1                                      00053890
        SXD     A9,1                                        00053900
        SXD     A12,1                                       00053910
        TXI     *+1,1,-2                                    00053920
        SXD     A7,1                                        00053930
        SXD     A36,1                                       00053940
        SXD     A18,1                                       00053950
        SXD     A38-1,1                                     00053960
        SXD     A21-1,1                                     00053970
        SXD     A23,1                                       00053980
```

```
        COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

        SXD    A25,1                                   00053990
        SXD    A31,1                                   00054000
        SCD    A51,1                                   00054010
        SCD    A51+1,1                                 00054020
        TXI    *+1,1,2                                 00054030
        CLA    6,4                                     00054040
        PAC    0,3                                     00054050
        CLA    7,4                                     00054060
        PAC    0,5                                     00054070
A51     TXI    *+1,3,2L -2                             00054080
        TXI    *+1,5,2L -2                             00054090
*                                                      00054100
*      NORMALIZATION OF ROWS                           00054110
*                                                      00054120
A2      AXT    2L,2                                    00054130
        PXA    0,3                                     00054140
        PAX    0,6                                     00054150
        PAX    0,7                                     00054160
        PXA    0,0                                     00054170
A3      LDQ    A,6                                     00054180
        LRS    0                                       00054190
        TLQ    *+2                                     00054200
        XCA                                            00054210
        LDQ    A+1,6                                   00054220
        LRS    0                                       00054230
        TLQ    *+2                                     00054240
        XCA                                            00054250
        TNX    A4,2,2                                  00054260
        TXI    A3,6,2N                                 00054270
A4      TZE    E1                                      00054280
        STO    T                                       00054290
        CLA    =1.0                                    00054300
        FDP    T                                       00054310
        STQ    T                                       00054320
        AXT    2L,2                                    00054330
A5      FMP    A,7                                     00054340
        STO    A,7                                     00054350
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
         LDQ     T                                           00054360
         FMP     A+1,7                                       00054370
         STO     A+1,7                                       00054380
         LDQ     T                                           00054390
         TNX     A6,2,2                                      00054400
         TXI     A5,7,2N                                     00054410
A6       PXA     0,5                                         00054420
         PAX     0,6                                         00054430
         AXT     LB,4                                        00054440
1A6      FMP     B,6                                         00054450
         STO     B,6                                         00054460
         LDQ     T                                           00054470
         FMP     B+1,6                                       00054480
         STO     B+1,6                                       00054490
         LDQ     T                                           00054500
         TNX     2A6,4,1                                     00054510
         TXI     1A6,6,2N                                    00054520
2A6      TNX     A7,1,2                                      00054530
         TXI     *+1,3,2                                     00054540
         TXI     A2,5,2                                      00054550
A7       TXH     B7,2,2L -2                                  00054560
*                                                            00054570
*       SEARCH FOR MAXIMUM-PIVOT IN COLUMN                   00054580
*                                                            00054590
         PXA     0,2                                         00054600
         PAX     0,1                                         00054610
         PXA     0,3                                         00054620
         PAX     0,6                                         00054630
         PXA     0,0                                         00054640
A8       LDQ     A,6                                         00054650
         LRS     0                                           00054660
         TLQ     *+3                                         00054670
         XCA                                                 00054680
         SXA     A10,1                                       00054690
         LDQ     A+1,6                                       00054700
         LRS     0                                           00054710
         TLQ     *+3                                         00054720
```

COMPLEX SIMULTANEOUS ECUATION SUBROUTINE

```
         XCA                                               00054730
         SXA    A10,1                                      00054740
         TXI    *+1,1,2                                    00054750
A9       TXH    *+2,1,2L                                   00054760
         TXI    A8,6,-2                                    00054770
         LDQ    TOL                                        00054780
         TLQ    *+2                                        00054790
         TRA    E3                                         00054800
A10      AXT    **,1                                       00054810
         SXD    *+1,2                                      00054820
         TNX    A17,1,**                                   00054830
*                                                          00054840
*       ROW INTERCHANGE                                    00054850
*                                                          00054860
         PXA    0,3                                        00054870
         PAX    0,6                                        00054880
         PAX    0,7                                        00054890
         SCD    *+1,1                                      00054900
         TXI    *+1,7,**                                   00054910
         PXA    0,2                                        00054920
         PAX    0,4                                        00054930
A11      CLA    A,6                                        00054940
         LDQ    A,7                                        00054950
         STO    A,7                                        00054960
         STQ    A,6                                        00054970
         CLA    A+1,6                                      00054980
         LDQ    A+1,7                                      00054990
         STO    A+1,7                                      00055000
         STQ    A+1,6                                      00055010
         TXI    *+1,4,2                                    00055020
A12      TXH    A13,4,2L                                   00055030
         TXI    *+1,6,2N                                   00055040
         TXI    A11,7,2N                                   00055050
A13      PXA    0,5                                        00055060
         PAX    0,6                                        00055070
         PAX    0,7                                        00055080
A14      AXT    LB,4                                       00055090
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
       SCD    *+1,1                        00055100
       TXI    *+1,6,**                     00055110
A15    CLA    B,7                          00055120
       LDQ    B,6                          00055130
       STO    R,6                          00055140
       STQ    B,7                          00055150
       CLA    B+1,7                        00055160
       LDQ    B+1,6                        00055170
       STO    B+1,6                        00055180
       STQ    B+1,7                        00055190
       TNX    A17,4,1                      00055200
A16    TXI    *+1,6,2N                     00055210
       TXI    A15,7,2N                     00055220
*                                          00055230
*     DIVISION OF ROW BY PIVOT             00055240
*                                          00055250
A17    NZT    A,3                          00055260
       TRA    B1                           00055270
       LDQ    A,3                          00055280
       FMP    A,3                          00055290
       STO    T                            00055300
       LDQ    A+1,3                        00055310
       FMP    A+1,3                        00055320
       FAD    T                            00055330
       STO    T                            00055340
       CLA    A,3                          00055350
       FDP    T                            00055360
       STQ    AN                           00055370
       CLA    A+1,3                        00055380
       FDP    T                            00055390
       STQ    AN                           00055400
A18    TXH    A21,2,2L -2                  00055410
       PXA    0,2                          00055420
       PAX    0,4                          00055430
       PXA    0,3                          00055440
       PAX    0,6                          00055450
A20    TXI    *+1,6,2N                     00055460
```

COMPLEX SIMULTANEOUS ECUATION SUBROUTINE

```
        LDQ     A,6                                     00055470
        FMP     AN                                      00055480
        STO     T                                       00055490
        LDQ     A+1,6                                   00055500
        FMP     AM                                      00055510
        FSB     T                                       00055520
        LDQ     A+1,6                                   00055530
        STO     A+1,6                                   00055540
        FMP     AN                                      00055550
        STO     T                                       00055560
        LDQ     A,6                                     00055570
        FMP     AM                                      00055580
        FAD     T                                       00055590
        STO     A,6                                     00055600
        TXI     *+1,4,2                                 00055610
        TXL     A20,4,2L -2                             00055620
A21     PXA     0,5                                     00055630
        PAX     0,6                                     00055640
        AXT     LB,4                                    00055650
1A21    LDQ     B,6                                     00055660
        FMP     AN                                      00055670
        STO     T                                       00055680
        LDQ     B+1,6                                   00055690
        FMP     AM                                      00055700
        FSB     T                                       00055710
        LDQ     B+1,6                                   00055720
        STO     B+1,6                                   00055730
        FMP     AN                                      00055740
        STO     T                                       00055750
        LDQ     B,6                                     00055760
        FMP     AM                                      00055770
        FAD     T                                       00055780
        STO     B,6                                     00055790
        TNX     *+2,4,1                                 00055800
2A21    TXI     1A21,6,2N                               00055810
*                                                       00055820
*      ROW REDUCTION                                    00055830
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
*                                            00055840
        PXA     0,2                          00055850
        PAX     0,1                          00055860
        PAX     0,4                          00055870
        TNX     A31,1,2                      00055880
        PXA     0,3                          00055890
        PAX     0,6                          00055900
        PAX     0,7                          00055910
        STA     A29                          00055920
        SXA     A26+1,5                      00055930
        SXA     3A26,5                       00055940
        TXI     *+1,6,2                      00055950
A22     TXI     *+1,7,2N                     00055960
        TXI     *+1,3,2N -2                  00055970
        SXA     A28,7                        00055980
A23     TXH     A26,2,2L -2                  00055990
        SXA     A27,3                        00056000
A24     LDQ     A,6                          00056010
        FMP     A,7                          00056020
        STO     T                            00056030
        LDQ     A+1,6                        00056040
        FMP     A+1,7                        00056050
        FSB     T                            00056060
        FAD     A,3                          00056070
        STO     A,3                          00056080
        LDQ     A,6                          00056090
        FMP     A+1,7                        00056100
        STO     T                            00056110
        LDQ     A+1,6                        00056120
        FMP     A,7                          00056130
        FAD     T                            00056140
        CHS                                  00056150
        FAD     A+1,3                        00056160
        STO     A+1,3                        00056170
        TXI     *+1,4,2                      00056180
A25     TXH     A27,4,2L -2                  00056190
        TXI     *+1,3,2N                     00056200
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
        TXI     A24,7,2N
A27     AXT     **,3
A26     AXT     LB,4
        AXT     **,7
        TXI     *+1,7,2
        SXA     *-2,7
1A26    LDQ     A,6
        FMP     B,5
        STO     T
        LDQ     A+1,6
        FMP     B+1,5
        FSB     T
        FAD     B,7
        STO     B,7
        LDQ     A,6
        FMP     B+1,5
        STO     T
        LDQ     A+1,6
        FMP     B,5
        FAD     T
        CHS
        FAD     B+1,7
        STO     B+1,7
        TNX     3A26,4,1
2A26    TXI     *+1,5,2N
        TXI     1A26,7,2N
3A26    AXT     **,5
        TNX     A29,1,2
A28     AXT     **,7
        PXA     0,2
        PAX     0,4
        TXI     *+1,3,2
        TXI     A23,6,2
A29     AXT     **,3
A31     TXH     A43,2,2L -2
        PXA     0,2
        PAX     0,1
```

COMPLEX SIMULTANEOUS ECUATION SUBROUTINE

```
        PAX   0,4
        PXA   0,3
        PAX   0,6
        PAX   0,7
A32     TXI   *+1,3,2N +2
        TXI   *+1,6,-2
A33     TXI   *+1,7,2N
        SXA   A37+1,5
        SXA   A41,5
        SXA   A40,3
        SXA   A39,7
A34     SXA   A38,3
A35     LDQ   A,6
        FMP   A,7
        STO   T
        LDQ   A+1,6
        FMP   A+1,7
        FSB   T
        FAD   A,3
        STO   A,3
        LDQ   A,6
        FMP   A+1,7
        STO   T
        LDQ   A+1,6
        FMP   A,7
        FAD   T
        CHS
        FAD   A+1,3
        STO   A+1,3
        TXI   *+1,4,2
A36     TXH   A37,4,2L -2
        TXI   *+1,3,2N
        TXI   A35,7,2N
A37     AXT   LB,4
        AXT   **,7
        TXI   *+1,7,-2
        SXA   *-2,7
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
1A37    LDQ     A,6                                    00056950
        FMP     B,5                                    00056960
        STO     T                                      00056970
        LDQ     A+1,6                                  00056980
        FMP     B+1,5                                  00056990
        FSB     T                                      00057000
        FAD     B,7                                    00057010
        STO     B,7                                    00057020
        LDQ     A,6                                    00057030
        FMP     B+1,5                                  00057040
        STO     T                                      00057050
        LDQ     A+1,6                                  00057060
        FMP     B,5                                    00057070
        FAD     T                                      00057080
        CHS                                            00057090
        FAD     B+1,7                                  00057100
        STO     B+1,7                                  00057110
        TNX     A41,4,1                                00057120
2A37    TXI     *+1,5,2N                               00057130
        TXI     1A37,7,2N                              00057140
A41     AXT     **,5                                   00057150
        TXI     *+1,1,2                                00057160
        TXH     A40,1,2L -2                            00057170
A38     AXT     **,3                                   00057180
A39     AXT     **,7                                   00057190
        PXA     0,2                                    00057200
        PAX     0,4                                    00057210
        TXI     *+1,3,-2                               00057220
        TXI     A34,6,-2                               00057230
A40     AXT     **,3                                   00057240
        TXI     *+1,5,-2                               00057250
        TXI     A7,2,2                                 00057260
A43     CLA     =1                                     00057270
        TRA     MSIMEC+1                               00057280
B1      STZ     AM                                     00057290
        CLA     =1.0                                   00057300
        FDP     A+1,3                                  00057310
```

COMPLEX SIMULTANEOUS EQUATION SUBROUTINE

```
         STQ     AN                                              00057320
         TRA     A18                                             00057330
*                                                                00057340
*       BRANCH FOR LAST ROW                                      00057350
*                                                                00057360
B7       CLA     A,3                                             00057370
         SSP                                                     00057380
         LDQ     TOL                                             00057390
         TLQ     A17+2                                           00057400
         CLA     A+1,3                                           00057410
         SSP                                                     00057420
         TLQ     A17                                             00057430
*                                                                00057440
*       ERROR BRANCHES                                           00057450
*                                                                00057460
E3       CLA     =2                                              00057470
         TRA     MSIMEC+1                                        00057480
E1       CLA     =3                                              00057490
         TRA     MSIMEC+1                                        00057500
*                                                                00057510
*       STORAGE                                                  00057520
*                                                                00057530
TOL      OCT     151400CC0000                                    00057540
AM                                                               00057550
AN                                                               00057560
T                                                                00057570
A        EQU     0                                               00057580
B        EQU     0                                               00057590
2L       EQU     0                                               00057600
2N       EQU     0                                               00057610
LB       EQU     0                                               00057620
         END                                                     00057630
```

APPENDIX IV. SAMPLE DATA SHEETS

The following pages are sample data sheets for a computer run on three modes at three frequencies. The potential will not be computed for the first mode. The generalized forces found will be L_{21}, L_{22}, L_{23}, L_{31}, L_{32}, L_{33}.

Of the first fourteen cards, the cards numbered 6, 9, 10, 11, 12, 13, 14 do nothing and are included only to indicate how all data is entered. Cards 1 through 14 are complete in this respect, and all later cards are of the same type as one of the first fourteen. The data used in the least squares surface fit for the deflection is entered in locations 101 through 700.

Card number 22 represents 56 cards for the intermediate data points which would have to be included in an actual run.

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A L P H A 1		title - columns 13 - 72
13	6 0 D E G R E E C R		(optional)
25	Ø P P E D D E L T A		
37	A T 2 0 C P S		
49		73 80	
61		0 1	
1	A L P H A 1 3		mode title - columns 13 - 72
13	P L U N G E		(optional)
25			
37			
49		73 80	
61		0 2	
1	2 3		
13	2 0 . 0		frequency, ν use consistent
25	2 5 . 0		root chord length, b units here and
37	1 2 8 7 5 . 0		speed of sound, a in items 30 - 36
49		73 80	Indicator for Subsequent frequencies
61		0 3	
1	2 7		
13	3 0		no of boxes along root chord, L
25	3		no. of modes to be given
	1		no. of sections of leading edge given, NS
		73 80	
		0 4	

FORM 114-C-17 REV. 7-60 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH	
1	3.0			coordinates of points on leading edge
11	0.0		$\tilde{y}_0$	
21	15.60		$\tilde{x}_1$	
31	9.0		$\tilde{y}_1$	
41		73 80	$\tilde{x}_2$	used if NS ≥ 2
61		0 5	$\tilde{y}_2$	
1	3.5			
11			$\tilde{x}_3$	used if NS = 3
21			$\tilde{y}_3$	
31				
41		73 80		
61		0 6		
1	3.9			
11	1		G	non zero value suppresses calculation of potential
21				for this mode
31				
41		73 80		
61		0 7		
1	− 4.6			
11	1.0		d_{00}	beginning of list of coefficients of deflection series
21			d_{10}	$\sum d_{mn} x^n y^m$
31			d_{20}	
41		73 80	d_{30}	
61		0 8	d_{40}	

FORM 114-C-17 REV. 7-65 · VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ________ PROGRAMMER ________ DATE ________ PAGE ____ of ____ JOB NO. ________

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	5 1		
13			d_{01}
25			d_{11}
37			d_{12}
49		73 80	etc
61		0 9	
1	8 7		
13			indicator for deflection coefficient printout
25			indicator for upwash array printout
37			indicator for potential array printout
49		73 80	indicator for potential coefficient printout
61		1 0	indicator for potential and pressure printout
1	9 8		
13			NP number of points at which deflection is given
25			NX number of x values
37			NY number of y values
49		73 80	
61		1 1	
1	1 0 1		
13			$\hat{x}_1$ data for first point
25			$\hat{y}_1$
37			$f(\tilde{x}_1,\tilde{y}_1)$
49	.	73 80	weight (optional - weight 1.0 used if no entry is made)
61		1 2	

FORM 114-C-17 REV. 7-5. VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 0 5		
13			$\tilde{x}_2$ data for second point
25			$\tilde{y}_2$
37			$f(\tilde{x}_2,\tilde{y}_2)$
49		73 80	weight
61		1 3	
1	1 0 9	2	
13			$\tilde{x}_3$ data for third point
25			$\tilde{y}_3$
37			etc
49		73 80	
61		1 4	
1	A L P H A 1 3		
13	P I T C H A B O U T		beginning of data for second mode
25	X = 0		
37			
49		73 80	
61		1 5	
1	3 9		
13	0		zero cancels previous value, causing potential to be computed
25			
37			
49		73 80	
61		1 6	

FORM 114-C-17 REV. 7-60 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ______

		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		8 7		
13		1		
25		1		
37		1		
49		1		
61		1	1 6	
1	–	4 6		last card for second mode
13		0 . 0		this entry cancels previous value of d_{10}
25		0 . 0 4		
37				the polynomial is 0.04Y, normalized for maximum value 1.0
49				
61			1 8	
1		A L P H A 1 3		
13	F	I R S T B E H D I N		beginning of data for third mode
25	G	M O D E		
37				
49				
61			1 9	
1		9 8		
13		5 8		
25		8		
37		8		
49				
61			2 0	

FORM 114-C-17 REV. 7-60 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.________ PROGRAMMER____________ DATE________ PAGE____ of____ JOB NO.________

Col.		NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		101		
11		23.6		data for first point
21		0.5		
31		0.0235		
41			73 ... 80	
51			21	
1		105		
11		960		
21				
31				
41			73 ... 80	
51			22	
1	—	331		data for last point; last data for this mode
11		5.8		
21		0.6		
31		0.01286		
41			73 ... 80	
51			23	
1	—	33		data for a second frequency
11		10.0		new frequency
21				
31				
41		1	73 ... 80	indicator
51			24	

FORM 114-C-17 REV. 7-60 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ________ PROGRAMMER ________________ DATE ________ PAGE ______ of ______ JOB NO. ____________

Column	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2.3		data for a third frequency
13	40.0		
25			
37			
49		73 80	
61		2 5	
1			
13			
25			
37			
49		73 80	
61			
1			
13			
25			
37			
49		73 80	
61			
1			
13			
25			
37			
49		73 80	
61			